Managing Patient Panels with Non-Physician Providers

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In recent years, the drive to contain health care costs has increased scrutiny of the traditional mode of delivering primary care where a patient is treated only by his primary care physician. In particular, greater reliance on non-physician providers has been suggested as a lower-cost alternative to the traditional set-up. In this paper, we consider a homogeneous patient panel treated by a solo primary care physician and develop a new model of patient health dynamics in which the health state for each patient on the physician’s panel follows Markovian transitions between “healthy”, “intermediate”, and “sick” states. In contrast to most currently used models, we treat patient demand for office visits as endogenous and managed by a physician via selection of a revisit frequency consistent with patient preferences. We model these preferences for the frequency of office visits using patients’ perception of their health status as well as the disutility associated with falling sick. At the center of our analysis are the interconnected decisions that a physician makes regarding the size of her patient panel and the patient revisit frequency. Our results quantify the overall impact of non-physician providers on physician’s choices, physician’s expected daily compensation, and patients’ health. We characterize care settings, defined in terms of care effectiveness, characteristics of patient panel, as well as physician’s compensation scheme, that result in both parties, physician and patients, being better off as well as settings where at least one of the parties is worse off compared to the traditional approach.

Key words: Non-Physician Providers, Health Care, Primary Care, Stochastic Models, Service Operations

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1. Introduction

Primary care is the backbone of any health care system and a major point of access to care for most patients. Each year, physicians hold about 500 million primary care office visits in the United States (Rui et al. 2017). A rapidly aging population combined with legislative expansions to healthcare coverage, namely via the Affordable Care Act, are expected to induce additional primary care demand in coming years. To keep up with these trends and to control rising healthcare costs, many health systems are leveraging alternative modes of primary care delivery. The standard way of delivering primary care is through interactions between a patient and his primary care physician during an office visit. We will use the term “traditional” to describe this care delivery mode. One proposed alternative to this approach is to use “non-physician providers” for care delivery (Kaiser Family Foundation 2015). Under this mode, patient care during office visits is partially delegated to a non-physician provider, such as a nurse practitioner or a physician assistant.

Assuming that high quality care can be delivered using non-physician providers, such an innovation in the delivery of primary care appears promising. Holding the number of times a patient receives primary care services constant, one would expect that the higher efficiency of this new delivery form would let a primary care provider see more patients. The problem, however, is that we have no reason to believe that doctors and patients would not alter their behavior in response to this new form of care delivery. Doctors determine the frequency with which they see patients based on a set of medical and economic factors. Similarly, patients balance the costs and benefits of consulting with the primary care provider. If and to what extent the resulting equilibrium outcome leads to larger patient panels, higher physician earnings, and healthier patients is thus an interesting question.

To answer this question, we present a new approach to analyzing the impact of alternative primary care delivery modes by treating patient demand as an endogenous process governed by both patient preferences and physician financial incentives. In resolving patient and physician trade-offs, a key role is played by the dynamics of patient health under a particular primary care delivery mode. In our analysis, we model these dynamics using a Markovian continuous-time setting in which patients transition between “healthy” (H), “intermediate” (I), and “sick” (S) states. Once in the “sick” state, the patient schedules a same-day office visit. These rates can be interpreted as overall rates of decline in patient health.

In our analysis, we consider homogeneous patient panels characterized by the health decline rates, the disutility of getting sick, and the flexibility regarding their preferences for the frequency of office visits. We use the term patient revisit interval, or RVI in short, to describe the number of days between scheduled office visits. Based on the values of the health decline rates, we characterize patient panels as being “low-risk” (low rate of health decline) or “high-risk” (high
rate of health decline). Similarly, based on the value of disutility associated with being sick, we designate patient panels as being “stoic” (low disutility) or “worried” (high disutility). Finally, based on the flexibility of RVI preferences, we distinguish between “flexible” and “non-flexible” panels. From the perspective of physician compensation incentives, we consider “fee-for-service” physicians receiving a fixed fee for each office visit, and “capitation” physicians receiving a fixed fee for each patient on her panel. In our model, non-physician providers are responsible for some of the office visits, providing a mechanism for partial diversion of patient demand for care away from physician’s capacity.

Our modeling approach allows us to establish the following results:

1. We derive the daily probabilities of unscheduled patient office visits (Propositions 1 and 2) and preferred values of office revisit intervals (Proposition 3) in settings with and without non-physician providers. We also show how patient response to the presence of non-physician providers depends on the disparity of the transition rates between patient health states (Corollary 1).

2. We show that, under the traditional care delivery mode, physicians compensated on a fee-for-service basis choose to see their patients more often than physicians compensated on a capitation basis. As a result, fee-for-service physicians will select smaller panel sizes as compared to capitation physicians (Proposition 4).

3. When non-physician providers are used, patient health improves except in the fee-for-service setting where non-physician providers save a considerable amount of physician’s time, and patients accept a wide range of RVIs (Propositions 5 and 6).

4. We show that fee-for-service physicians always earn more as compared to the traditional care delivery mode. However, capitation physicians earn more if the care delivered by non-physician providers is of high quality and offers significant time savings for physicians (Propositions 5 and 7).

Our paper is organized as follows. Section 2 reviews the relevant literature; section 3 introduces our model of patient health dynamics under two modes of primary care delivery, while section 4 presents analysis of the implications of enhancing the traditional approach via non-physician providers. We consider a numerical example based on realistic parameter values in section 5 and discuss our findings in section 6.

2. Literature Review

Our analysis is related to several literature streams. In the medical research literature, use of non-physician providers (hereafter NPPs) have been the subject of a growing number of studies (Naylor and Kurtzman 2010). Use of nurse practitioners in primary care dates back to the 1960s, partially
caused by a shortage of primary care physicians in rural areas. However, the scope of practice for these providers has been expanding recently, and its further expansion is a source of active debate. Nurse practitioners and physicians go through very different training programs, and the extent to which they can be used as substitutes in primary care is far from established (Baron and Cassel 2008, Liu and D’Aunno 2012). Dobson et al. (2009) show that the use of non-physician service providers can lead to substantial “coordination costs” that may outweigh the beneficial impact of potential increase in physician’s service capacity. White et al. (2017) compare three workflow modes for “mid-level” providers who are similar to NPPs in our study: (1) “physician only” (no mid-level provider), (2) “ice-breaker” (mid-level provider seeing patients before the physician to reduce the physician’s service time), and (3) “standalone” (mid-level provider fully substituting the physician for some of the patient visits). The authors show that all three modes can be financially optimal depending on the operational and financial conditions of the clinic, but the “ice-breaker” mode is the least likely to be optimal. Also, using data from nursing homes, Lu et al. (2017) show that automation technology can increase or decrease nurse staffing levels depending on a facility’s position within its competitive market.

The argument for further involvement of non-physician providers is accentuated by the potential shortage of primary care physicians in the United States suggested to emerge under the traditional care delivery mode (Mitka 2007). Green et al. 2013 provide analysis that suggests that primary care delivery modes based on e-visits and non-physician providers, combined with the spread of shared practices (i.e., physicians accepting joint responsibility for patient care), can reduce or even eliminate this projected shortage.

In the operations literature, there is a growing body of work on primary care, and several papers focus on panel sizing in particular. For example, Green et al. (2007b) and Ozen and Balasubramanian (2013) provide analyses to show how physicians can select panel sizes accounting for risk adjustment. Two other papers focus on the question of how missed appointments (patient no-shows) affect panel sizing (Green and Savin 2008, Liu and Ziya 2014), and another related paper examines how panel sizing affects the number of daily appointment slots that a physician should offer to minimize backlogs (Zacharias and Armony 2016). We also build on the prior literature in appointment scheduling (LaGanga and Lawrence 2007, Begen and Queyranne 2011, Zacharias and Pinedo 2014) and include the probability of the provider having to work overtime as a feature in our model. A key contribution of the present analysis is the modeling of patient demand for care. Prior papers treat this demand as exogenous, but we endogenize the process such that physicians must select RVI values consistent with patient preferences.

The model of patient health evolution we develop resembles the dynamics in machine interference/repairman models (reviewed in Haque and Armstrong 2007), with the patient playing the role...
of the “machine” on which physician performs “repairs”. The key difference between the modeling approaches in this literature and the one we use is the way we treat patient responses to non-physician providers: in contrast with machine interference/repairmen literature, we assume that patients actively re-evaluate “acceptable” RVI ranges when a new care delivery mode is introduced.

We also build on prior studies that examine the impact of various interventions in the primary care setting. In the operations management literature, Bavafa et al. (2018b) and Bavafa et al. (2018a) consider a setting with e-visits in primary care. Deo et al. (2013) study the problem of determining revisit intervals for asthma patients, and Ramdas and Darzi (2017) discuss the use of shared medical appointments. Relatedly, Balasubramanian et al. (2012) examines novel ways to redesign primary care delivery including physicians caring for other physicians’ patients when needed so that patients can receive improved access and continuity of care.

Our work builds on these papers by studying a different change in delivery mode that is complementary to the ones discussed in these studies. Of the mentioned studies, our work is closest to Bavafa et al. (2018b) where the authors study patient panel size and revisit interval choices by treating both physicians and patients as active entities reacting to the changes in the delivery system based on the trade-offs they face. The key difference between our work and Bavafa et al. (2018b) is that our model is focused on an entirely different intervention by looking at non-physician care, while Bavafa et al. (2018b) studies e-visits in primary care. Aside from this key difference, which has significant impacts on the analysis, the two studies differ along several additional dimensions. First, the NPP mode in the present analysis operates by impacting patient health, whereas the e-visit mode studied in Bavafa et al. (2018b) operates by impacting patient costs. In that paper, e-visits have no direct impact on patient health. This difference, combined with the stochasticity of health in the present model, leads to different analytical models and insights. Second, we explicitly model the stochastic arrival of patients; the physician must account for this or face working overtime. As a result, the physician in our model resists choosing very large RVI values or large panel sizes. We allow for overtime because prior work in appointment scheduling has highlighted the importance of considering overtime costs. By modeling this feature, the present model is able to capture the extent to which NPPs alter the distribution parameters of patient arrivals. By contrast, the physician modeled in Bavafa et al. (2018b) only needs to ensure that the average supply of appointments meets their average demand. Third, because the present paper’s focus is on an intervention that impacts patient health, we explicitly model patient health with a three-state Markov chain with continuous revisit intervals. By contrast, Bavafa et al. (2018b) modeled patient health with one probability parameter corresponding to a two-point discrete distribution of revisit intervals. Allowing a continuous support for patient revisit intervals as we do in the present paper,
along with a richer model of patient health, allows us to explore more realistic comparative statics regarding patient health.

Physician compensation scheme is one of the main factors that determine how physicians react to changes in the care delivery modes. Both the economics and the operations literature include a number of studies focusing on the effects of monetary incentives (Hickson et al. 1987, Stearns et al. 1992, Gosden et al. 2000, Lee et al. 2010, Shumsky and Pinker 2003) and non-monetary incentives (Song et al. 2017) on physician behavior. In particular, a number of recent papers in the operations management literature have focused on the impact of fee-for-service and capitation contracts on patient care (Andritsos and Tang 2018, Adida et al. 2016, Adida and Bravo 2018).

A separate group of papers have been focused on the way that the patient revisit interval is chosen in practice. Among the factors influencing the selection of the RVI value, those most frequently cited are the perceived patient health state (Welch et al. 1999), the necessity to follow-up on test results (DeSalvo et al. 2000), and the need to evaluate the impact of changes in patient treatment (Schwartz et al. 1999). It is important to note that physician practice style has been found to be an important determinant of the choice of RVI, even after controlling for patients’ characteristics. Schectman et al. (2005) provides the following summary of these findings: “Provider practice style may reflect scheduling habits acquired from previous training independent of the patients’ medical needs. For example, providers have often been trained to schedule their patients every 3 or 4 months routinely, regardless of disease severity.” The authors show that a physician education program could lead to increased RVI values.

3. Modeling Patient Health Dynamics, Preferences for Revisit Intervals, and Physician Compensation

In this section, we describe patient health dynamics under two modes of delivering primary care: “traditional” mode with physician-only care and “non-physician provider” mode with partial use of non-physician providers. Next, we use patient health dynamics model to analyze patient preferences for revisit interval values. In the last step, models of patient health dynamics and preferences for revisit intervals are embedded into the physician revenue maximization problem.

3.1. Evolution of Patient Health Under Alternative Modes of Primary Care Delivery

We model the evolution of patient health status using a continuous-time Markov chain, assuming that the health state dynamics for any patient on physician’s patient panel is independent from and identical to that of any other patient on the same panel. In our model, patient health status undergoes transitions between three health states: $H$ (“healthy”), $I$ (“intermediate”), and $S$ (“sick”). We use a three-dimensional vector to describe the state of patient health, $Q_t = [Q_t^H, Q_t^I, Q_t^S]^T$ at
any time $\tau \geq 0$, with three components $Q^k_\tau$, $k = H, I, S$ describing the probability of a patient to be in the state $k$ at time $\tau$. Below we describe our model of patient health dynamics under each of the two approaches to care delivery: traditional and non-physician provider modes.

### 3.1.1. Patient Health Dynamics Under Traditional Mode of Care Delivery

Under the traditional mode of care delivery, a patient requests service from his primary care physician either through regularly scheduled visits every $r$ days, or when a patient falls sick. We assume that the evolution of patient health is Markovian, and that, in absence of any scheduled office visits

$$
\frac{dQ^k_\tau}{d\tau} = Q^k_\tau P^t,
$$

where transitions rate matrix $P^t$ is given by:

$$
P^t = \begin{bmatrix}
-p_1 & 0 & \omega \\
p_1 & -p_2 & 0 \\
0 & p_2 & -\omega
\end{bmatrix},
$$

where $p_1 \ll 1$ and $p_2 \ll 1$ are transition rates (measured in the units of 1/day) between the $H$ and the $I$ states and the $I$ and the $S$ states, respectively. In our model, a patient in the health state $H$ makes a transition to the health state $I$ after a random, exponentially distributed duration with an expected value of $1/p_1$ days. Once in the state $I$, a patient becomes sick after spending a random, exponentially distributed duration with the expected value of $1/p_2$ days. Once a patient is sick, he arrives at his physician’s office, receives care, and is “restored” to the state $H$ after an exponential random duration characterized by the rate $\omega \gg 1$. The exponentially distributed durations that a patient spends in the states $H$, $I$ and $S$ are assumed to be independent. From the point of view of patient health dynamics, the treatment process is nearly instantaneous (i.e., $\omega \gg p_1$ and $\omega \gg p_2$).

In our analysis of all care delivery modes we assume that a physician operates under the “open access” paradigm (Murray and Tantau 2000), so that patients have same-day access to care in case of need and no patient backlog is formed. This assumption allows for a tractable analysis of patient panel size and office revisit interval selection. As a consequence, in the presence of uncertain daily demand for care a physician is required to work “overtime” on days when patient demand exceeds nominal service capacity.
The Markov chain dynamics identified by the transition rate matrix (1) is illustrated in Figure 1. In the absence of scheduled office visits, for \( \omega \to \infty \), the health state dynamics described by (1) will result in the stationary state where, for \( p_1 = p_2 = p \), a patient spends half of his time, in expectation, in the \( H \) state and the other half in the \( I \) state. When office visits are scheduled every \( r \) days, the overall health state dynamics changes. In particular, every \( r \) days a patient makes an office visit and, if necessary, is “restored” to the \( H \) state. Thus, Markovian system describing patient’s health state evolves in a periodic fashion, starting every \( r \)-day period in the health state \( Q_0 = [1,0,0]^T \).

As shown in Section 3.2, the rate for a patient to fall sick and generate an unscheduled office visit on a particular day plays a key role in the analysis of patient preferences for the RVI values as well as physician’s choice of patient panel size and RVI. We denote this rate as \( \pi_U \), where the subscript \( U \) stands for an “unscheduled” visit. Since an unscheduled visit will be generated, at the rate \( p_2 \), once a patient is in the state \( I \), for \( p_2 \ll 1 \), \( \pi_U \) can be expressed as \( p_2 \bar{Q}_I \), where \( \bar{Q}_I \) is an average (over an \( r \)-day cycle) probability of a patient to be in the \( I \) state. The following result provides a closed-form expression for \( \pi_U \) under the traditional mode in the limit of \( \omega \to \infty \).

**Proposition 1.** Under the traditional mode of primary care delivery with the office revisit interval of \( r \) days, the average (over an \( r \)-day cycle) probability for a patient to generate an unscheduled office visit on any day is

\[
\pi_U^t(p_1, p_2, r) = \frac{p_1 p_2}{p_1 + p_2} \left( 1 - \frac{1 - e^{-(p_1+p_2)r}}{(p_1+p_2)r} \right). \quad (2)
\]

As expected, the probability of generating an unscheduled office visit in (2) is an increasing function of both the daily health state decline rates \( p_1 \) and \( p_2 \) and the chosen RVI value \( r \). As the RVI value increases, \( \pi_U^t(p_1, p_2, r) \) approaches \( \frac{p_1 p_2}{p_1 + p_2} \), which reflects the fact that without scheduled office visits, the average time until becoming sick for a patient in state \( H \) is \( \frac{1}{p_1} + \frac{1}{p_2} \) days. Also, for \( p_1 = p_2 = p \) and a fixed \( r \), when \( p \to 0 \), \( \pi_U^t(p, p, r) = r^2 p^2 + o(p^2) \), an expression that reflects the probability \( \frac{r^2}{2} \) for a patient to generate exactly one office visit between the “restarts” of patient’s health state dynamics every \( r \) days.

We model patient health dynamics using a three-state Markov chain to reflect the reality that patients are more likely to become sick as time since the last office visit increases. That is, the stochastic process of falling sick has an increasing failure rate (IFR). Because of this IFR property, the probability of becoming sick is an increasing function of \( r \), and the patient can reduce the value of \( \pi_U^t \), by decreasing \( r \). A Markov model with two states (i.e., “healthy” and “sick” states only) will produce memoryless transitions between the \( H \) and \( S \) states. Therefore, in such a model, the probability of getting sick in each period stays the same regardless of the frequency of scheduled office visits, and the scheduled office visits carry no utility for the patient. In other words, the
stochastic process of falling sick in a two-state Markov chain has a constant failure rate, and the patient’s probability of getting sick is independent of \( r \).

Also, note that the focus of our model is not on clinical distinctions between different chronic diseases, but, rather, on the dynamics of generation of unscheduled office visits, irrespective of the specific underlying clinical reason. Therefore, \( H, I, \) and \( S \) states in our model reflect the operational rather than clinical states that create demand on the supply of physician’s care capacity. In other words, from an operational standpoint, \( H, I, \) and \( S \) states can each combine rather different clinical states across various chronic conditions, with the commonality being the demand rates they generate.

3.1.2. Patient Health Dynamics in the Presence of Non-Physician Providers: Using non-physician providers enables a patient to receive care while spending less face-to-face time with his physician. However, it may also reduce the quality of provided care since physicians are better-trained care providers. We model this potential quality reduction as follows: if a patient falls sick and generates an unscheduled office visit, his care is delivered, at least partially, by a non-physician provider, and the patient is “restored” to the \( H \) state with probability \( \gamma \leq 1 \). On the other hand, we assume that during a scheduled office visit patient care is provided by a physician, and such a visit “restores” the patient to the state \( H \) with probability 1. The presence of non-physician providers does not alter patient health decline dynamics, but, rather, allows for partial restoration of patient health status once he gets sick. A key element of the care mode based on the use of non-physician providers is the diversion of some of the demand for office care away from physician’s service capacity.

Under these assumptions, the matrix describing transition rates between patient health states in the absence of scheduled visits is given by

\[
P^n = \begin{bmatrix}
-p_1 & 0 & \omega \gamma \\
p_1 & -p_2 & \omega(1 - \gamma) \\
0 & p_2 & -\omega
\end{bmatrix}.
\] (3)

The Markov chain dynamics governed by (3) is illustrated in Figure 2: under this mode of care, a patient ends up in the \( I \) state after an unscheduled visit with probability \( 1 - \gamma \), and in the \( H \) state with probability \( \gamma \).

As expected, (3) reduces to (1) when \( \gamma = 1 \). Using (3), we can obtain a closed-form expression for the stationary probability of a patient falling sick when scheduled visits every \( r \) days are added to the model in the limit of \( \omega \to \infty \).

**Proposition 2.** Under the non-physician provider mode with revisit interval set to \( r \) days, the average (over an \( r \)-day cycle) probability for a patient to generate an unscheduled office visit on any day is

\[
\pi^n_U(p_1, p_2, r, \gamma) = \frac{p_1 p_2}{p_1 + \gamma p_2} \left( 1 - \frac{1 - e^{-(p_1 + \gamma p_2)r}}{(p_1 + \gamma p_2)r} \right). \tag{4}
\]
Figure 2 The Markov evolution of patient health in the absence of scheduled visits under the non-physician provider mode.

\[ \pi_n^U(p_1, p_2, r, \gamma) \] is a decreasing function of the effectiveness of non-physician provider visits, \( \gamma \), and an increasing function of \( p_1 \), \( p_2 \) and \( r \). When \( r \to \infty \) for fixed \( p_1 \), \( p_2 \) and \( \gamma \), \( \pi_n^U(p_1, p_2, r, \gamma) \to \frac{p_1 p_2}{p_1 + \gamma p_2} \), reflecting the fact that, in the absence of scheduled office visits, a patient in state \( H \) falls sick after \( \frac{\gamma p_1}{p_1} + \frac{1}{p_2} \) days on average. Also, for \( p_1 = p_2 = p \) and a fixed \( r \), when \( p \to 0 \), \( \pi_n^U(p, p, r, \gamma) = r \frac{p^2}{r} + o(p^2) \).

If the quality of care provided by NPPs is identical to the physician, then (4) converges to (2), i.e., as \( \gamma \to 1 \), \( \pi_n^U(p_1, p_2, r, \gamma) \to \pi_t^U(p_1, p_2, r) \). Also, if the NPPs are not able to restore patients to the healthy state (i.e., \( \gamma \to 0 \)), for \( r \to \infty \), \( \pi_n^U(p_1, p_2, r, \gamma) \to p_2 \). In such a case, the patient never comes back to the healthy state after leaving it for the first time and generates unscheduled office visits with rate \( p_2 \).

3.2. Physician Revenue Maximization and Patient Preferences for Office Revisit Intervals

In our model, a physician chooses her panel size, i.e., the number of patients she cares for. We assume there always exist patients looking for primary care providers and so there exists sufficient patient demand. When choosing her panel size, a physician is influenced by her compensation scheme. Also, the physician needs to see patients on her panel at intervals that are consistent with patient health preferences. We consider the impact of patient preferences on RVI first, and then proceed to formulate physician revenue maximization problem.

3.2.1. Modeling Patient Preferences for Office Revisit Intervals: It is well known that patients play an important role in selecting office revisit interval values (Welch et al. 1999). In our model, a patient determines an acceptable range for his office revisit frequency values, and his physician’s choice of the revisit interval must fall within this range. We consider a homogeneous panel of patients characterized by the daily probabilities \( p_1 \) and \( p_2 \) of decline in patient health state from state \( H \) to \( I \) and \( I \) to \( S \), respectively, and a “disutility”/cost \( c \) associated with falling sick. Patient’s objective is to select a positive RVI value \( r \) to minimize the expected daily cost under the care delivery mode \( m = t, n \).
\[ D = \frac{1}{r} + (1 + c)\pi^m_{U}, \]  

(5)

where \( t \) and \( n \) denote “traditional” and “non-physician providers”, respectively. The first term in (5) expresses daily cost associated with appointments scheduled every \( r \) days (we normalize the cost for each visit, including the co-pay, transportation cost, etc., to 1). The second term reflects the expected daily cost associated with falling sick and making an unscheduled office visit, with \( \pi^m_{U} \) being a daily probability of falling sick under the care delivery mode \( m = t, n \). Note that \( \pi^m_{U} \) is a function of \( p_1, p_2 \) and \( r \), and, depending on the chosen mode of care delivery, other parameters such as non-physician provider service effectiveness \( \gamma \). Since \( c \) denotes patient disutility of falling sick (above and beyond the cost associated with getting to the physician’s office), we can characterize patients with low value of \( c \) as “stoic” and the ones with high value of \( c \) as “worried”.

The global minima of (5), \( r^m_{o} \), for the two care delivery modes \( m = t, n \) are characterized as follows:

**Proposition 3.**

\[ r^t_{o} = -\frac{1}{p_1 + p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + p_2)^2}{(1 + c)p_1p_2} - 1 \right) \right) \right), \quad c > \frac{(p_1 + p_2)^2}{p_1p_2} - 1, \]  

(6)

\[ r^n_{o} = -\frac{1}{p_1 + \gamma p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + \gamma p_2)^2}{(1 + c)p_1p_2} - 1 \right) \right) \right), \quad c > \frac{(p_1 + \gamma p_2)^2}{p_1p_2} - 1. \]  

(7)

where \( W_{-1} \) is the lower branch of the Lambert \( W \) function.

The lower branch of the Lambert \( W \) function (Corless et al. 1996) is defined on the interval \([-\frac{1}{e}, 0]\) and takes values in the range from \(-1\) to \(-\infty\). Note that the conditions on \( c \) in (6) and (7) ensure that the patient’s cost minimization problem has an interior solution. Both these conditions can be expressed in terms of the ratio \( \eta = \frac{p_1}{p_2} \). The expression in (6) is equivalent to \( c > (1 + \eta) \left( 1 + \frac{1}{\eta^2} \right) - 1 \geq 3 \); thus, (6) implies that \( \frac{p_1}{p_2} \) or \( \frac{p_2}{p_1} \) cannot be too large. Similarly, the expression in (7) can be written as \( c > \gamma \left( 1 + \frac{\gamma}{\eta} \right) \left( 1 + \frac{\gamma}{\eta^2} \right) - 1 \geq 4\gamma - 1 \).

Figures 3 and 4 illustrate the values of \( r^m_{o} \) for the two care delivery modes. Figure 3 shows the values of \( r^t_{o} \) as functions of the “sickness” cost parameter \( c \) for different values of the daily health decline rates \( p_1 = p_2 = p \). For illustration purposes, we have chosen \( p = 0.01 \) corresponding to the average time interval between unscheduled visits of \( 2/p = 200 \) days in the absence of scheduled visits. Figure 3 compares the revisit interval values for \( p = 0.01 \) with those for \( p = 0.005 \) and \( p = 0.02 \) (illustrating the cases of “low-risk” and “high-risk” patient panels). As expected, patients with higher “sickness” cost and higher health decline rates choose lower revisit intervals. Note that in both cases, as the “sickness” cost approaches 3, patients forgo the scheduled visit option, relying exclusively on care received during unscheduled visits. Figure 4 describes changes in revisit rates
Figure 3  Revisit interval values \( r^t_0 \) under traditional care mode as a function of the “sickness” cost \( c \) for different values of the daily health decline rates \( p_1 = p_2 = p \).

Figure 4  Revisit interval values \( r^u_0 \) under the non-physician provider mode as a function of the “sickness” cost \( c \) for different values of the effectiveness of care delivered by non-physician providers \( \gamma \) \((p_1 = p_2 = 0.01)\).

when non-physician providers are used to provide care during the unscheduled visits (in this figure, the curve with \( \gamma = 1 \) corresponds to the traditional mode of care). We observe that decreases in the effectiveness of care in unscheduled visits translate into higher probability of a patient falling sick on any day between scheduled visits, driving the patient to see his physician more often.

The expressions for \( \pi_U(p_1, p_2, r) \) and \( r^d_0 \) are symmetric in \( p_1 \) and \( p_2 \): holding all else constant,
both $\pi^T_U(p_1, p_2, r)$ and $r^n_t$ stay the same if we swap the values of $p_1$ and $p_2$. This is not the case for $\pi^n_U(p_1, p_2, r, \gamma)$ and $r^n_o$, however, as $\gamma$ introduces asymmetry into these expressions. Thus, swapping $p_1$ and $p_2$ can increase or decrease RVIs. Corollary 1 formalizes this observation.

COROLLARY 1. Consider two patient panels: panel $A$ with $p_{1,A} = \alpha$ and $p_{2,A} = \beta$, and panel $B$ with $p_{1,B} = \beta$ and $p_{2,B} = \alpha$. Then, the following results hold.

1) Under the traditional mode, $r^n_{o,A} = r^n_{o,B}$.
2) Under the non-physician provider mode, $r^n_{o,A} \leq r^n_{o,B}$ iff $\alpha \leq \beta$.

A similar observation can be made by looking at the condition on $c$ in (7). For the limiting case in which $\gamma = 0$, the condition in (7) becomes $c > \frac{p_1}{p_2} - 1$ which is satisfied for any $c$ if $p_1 \leq p_2$. In other words, if the NPPs are not able to restore patients to the healthy state during unscheduled visits and $p_1 \leq p_2$, then patients schedule preventive office visits for any $c$. If the $p_1 > p_2$, however, whether patients schedule office visits or not depends on the value of $c$.

Figure 5 shows the RVI values for different combinations of $p_1$ and $p_2$. In this figure, we plot “indifference curves”: the RVI values are the same for all combinations of $p_1$ and $p_2$ on each curve. For example, the curve on the top-right corner of Figure 5a shows the values of $p_1$ and $p_2$ that lead to $r^n_o = 60$. Figure 5a is plotted for $\gamma = 1$ which corresponds to the traditional mode, and Figure 5b describes the case in which NPPs are involved with $\gamma = 0.5$. As shown in Corollary 1, the curves in Figure 5a are symmetric with respect to $p_1$ and $p_2$, but this symmetry disappears in Figure 5b.
Consider two patient panels at $A$ and $B$ in Figure 5b: $p_{1,A} = p_{2,B}$, $p_{2,A} = p_{1,B}$, and $p_{1,A} > p_{2,A}$. The RVI value for patients in panel $A$ is 100 days, while the RVI value for patients in panel $B$ is about 70 days.

Proposition 3 describes patient preferences for office revisit intervals in terms of three parameters: the daily health decline rates $p_1$ and $p_2$ and the disutility/cost associated with being sick $c$. If patients knew all these three values with perfect precision, the outcome of patient RVI optimization problem under any of the care delivery modes would result in a single RVI value. In reality, however, patients are often willing to accept a range of RVI values. To model this observation, we assume that while the values of $p_1$ and $p_2$ are known to patients (as well as their physician), patients do not know their value of $c$ with certainty, and estimate that it can lie in the interval of possible values $[c(1-\Delta), c(1+\Delta)]$ with $\Delta \in [0,1]$. Thus, in our model patient panel is described by a quadruple $(p_1, p_2, c, \Delta)$, with the value of $\Delta$ designating the level of uncertainty in the impact of sickness on patients, or, equivalently, the level of flexibility that patients on the panel exhibit in accepting a range of RVI values. Similar to characterizing patients as “low-risk” vs. “high-risk” or “worried” vs. “stoic” based on the panel’s values of $p$ and $c$, we can add another dimension to panel description, “flexible” (high values of $\Delta$) vs. “non-flexible” (low values of $\Delta$).

Given this cost parameter uncertainty, acceptable values of RVI for a patient panel characterized by a quadruple $(p_1, p_2, c, \Delta)$ will be in the interval $[r^-_m, r^+_m]$, where $r^-_m = r^-_m(p_1, p_2, c(1+\Delta))$ and $r^+_m = r^+_m(p_1, p_2, c(1-\Delta))$. This interval of acceptable values serves as a limitation on the choices of RVI a physician can make. Figure 6 illustrates this interval under the traditional mode of care delivery for $p = 0.01$ and $c = 3.5$. Note that for non-flexible panels, the acceptable range of revisit rates is very narrow, reducing to a single value as $\Delta \to 0$. On the other hand, even a moderate degree of flexibility ($\Delta \approx 0.15$) results in this example in a wide acceptable revisit interval range - from under 200 to more than 800 days.

3.2.2. Physician Compensation Schemes and Appointment Capacity Allocation:
We consider a physician who has a nominal capacity of $s$ equal-size appointments each day and who serves a homogeneous panel of $N$ patients. Given the choice of care delivery mode $m = t, n$, and the value of revisit interval $r$, the probability of a patient on physician’s panel falling sick and generating an unscheduled office visit on a particular day is $\pi^m_U$. As pointed out above, this value depends on several parameters: $p_1$, $p_2$, the daily rates of patient health state decline; $r$, the RVI value; and the care effectiveness parameter $\gamma$ when care is delivered by a non-physician provider. Note that in our model, the physician selects an RVI based on $p_1$, $p_2$, and the patient’s acceptable range of RVIs. Because the patient determines this range using $p_1$, $p_2$, $c$ (the cost of getting sick), and $\Delta$, the physician effectively knows $c$ as well. The parameter $c$ is known to the patient due to
privately observed inputs such as the potential for lost wages. For example, a high-wage patient is likely to select a smaller RVI (due to a higher $c$) compared to an unemployed patient. Even though the physician does not observe $c$ or its inputs such as patient wages, the physician can infer $c$ from the patient’s selected range of acceptable RVI values.

Under the assumptions on the patient health dynamics in Section 3.1, the number of unscheduled office visits generated by a patient on the panel is described by a Poisson random variable with the rate $\pi^m_U$. To facilitate analytical tractability, we use the normal approximation for this Poisson distribution (Feller 1968); this normal approximation is widely used in operations management literature (see, e.g., Kolesar and Green 1998, Green et al. 2007a). The resulting normal random variable has the following mean and standard deviation:

$$
\mu = N \pi^m_U, \quad \sigma = \sqrt{N \pi^m_U}.
$$

(8)

In choosing the size of her patient panel $N$ and the revisit interval $r$, a physician is guided by her compensation scheme, and in our analysis we focus on two common ones: fee-for-service (FFS) and capitation (CAP). We assume that under the fee-for-service incentive scheme a physician is paid a fixed amount for each in-office patient visit. As a result, the expected daily compensation for a fee-for-service physician is proportional to

$$
\Pi^m_{FFS} = \frac{N}{r} + N \pi^m_U.
$$

(9)

Under the capitation scheme a physician is paid a fixed amount (per time period, e.g., a year) for each patient on her panel. Thus, a capitation physician focuses on maximizing the size of her patient panel $N$:

$$
\Pi^m_{CAP} = N.
$$

(10)
The ability of a physician to maximize her daily compensation is limited by her daily appointment capacity \( s \). We model this limitation as a constraint ensuring that the likelihood for a physician to work overtime, given the daily mix of scheduled and unscheduled visits, is bounded by a “reasonable” level \( \delta \). This capacity constraint can be expressed as:

\[
\frac{N}{r} + N\bar{\pi}_U^m + z_\delta \sqrt{N\bar{\pi}_U^m} \leq s, \quad (11)
\]

where \( z_\delta \) is the standard normal \( z \)-score associated with probability \( \delta \) and \( \bar{\pi}_U^m \) is defined for each mode of care delivery as follows. Under the traditional mode of care delivery \( \bar{\pi}_U^t = \pi_U^t \), where \( \pi_U^t \) is given by (2). The use of non-physician providers allows a physician to serve each unscheduled office visit using only a fraction of physician’s appointment slot. In our model, we treat this reduction in physician’s involvement by assuming that an unscheduled office visit takes up an entire physician’s appointment slot with probability \( \tau_n \), and requires no physician’s time with probability \( 1 - \tau_n \). In other words, \( \tau_n \) corresponds to the expected value of the fraction of physician’s appointment slot consumed by an unscheduled office visit. Thus, the daily rate of unscheduled office visits that take up a full appointment slot is \( \bar{\pi}_U^n = p_2\tau^n Q_I^n = \tau^n\pi_U^n \), where \( Q_I^n \) is the average time spent in state \( I \), and \( \pi_U^n \) is given by (4).

Note that \( z_\delta \) reflects the physician’s willingness to work overtime as a “lifestyle choice”. This parameter is assumed to be physician-specific and independent of her compensation structure. Such an assumption presents a potential limitation of our analysis, as, in practice, physicians may adjust their preferences for overtime work depending on their compensation.

Based on our modeling approach, the problem of selecting patient panel size \( N \) and the office revisit interval \( r \) that a physician faces under the compensation scheme \( j = FFS,CAP \) and the care delivery mode \( m = t, n \) is

\[
\max_{r,N} \Pi_j^m
\quad \text{s.t.} \quad \frac{N}{r} + N\bar{\pi}_U^m + z_\delta \sqrt{N\bar{\pi}_U^m} \leq s, \quad r^m_- \leq r \leq r^m_+ \quad (12)
\]

where the \( r^m_- \) and \( r^m_+ \) are global minima of (5) for \( c(1 + \Delta) \) and \( c(1 - \Delta) \), respectively. Note that in our model the non-physician provider has a fixed cost. Therefore, to obtain physician profit in the case of \( m = n \), the fixed cost of hiring a non-physician provider needs to be included.

Let \( \left( \hat{r}_j^m, \hat{N}_j^m \right), j = FFS,CAP, m = t, n, \) be the optimal solution to (12). We also denote

\[
\hat{\pi}_{U,j}^m = \pi_U^m(p_1, p_2, \hat{r}_j^m), \quad \hat{\pi}_{U,j}^m = \pi_U^m(p_1, p_2, \hat{r}_j^m), \quad (13)
\]

for convenience. Below we analyze and compare the optimal solutions to (12) for the two care delivery modes.
4. Key System Outcomes Under Alternative Modes of Care Delivery

In this Section we focus on identifying the impact of non-physician providers on the patient panel size, revisit interval, as well as on the physician’s expected daily compensation and patient health levels.

4.1. Panel Size and RVI Decisions Under the Traditional Mode

Under the traditional approach to care delivery, the optimal revisit interval and panel size values can be characterized in closed form under three mild conditions on problem parameters: we consider a physician with the daily number of appointments \( s \) satisfying

\[
s \geq 0.5z_\delta^2,
\]

and a patient panel characterized by the daily health decline rates

\[
p_1, p_2 < 0.25,
\]

and the cost parameter \( c \) and the flexibility parameter \( \Delta \) such that

\[
c(1 - \Delta) > \frac{(p_1 + p_2)^2}{p_1 p_2} - 1.
\]

The analytical results that follow will rely on the sufficient conditions in (14)-(16). These conditions are likely to be satisfied in any realistic setting. For example, selecting \( \delta \) to be equal to a “near certainty” value of 0.975 corresponds to \( z_\delta = 1.96 \), so that (14) will hold for any physician that has at least \( s = 2 \) daily appointment slots. Also, (15) requires that the expected time it takes a patient to “descend” from one state to the next one (e.g., from the \( H \) state to the \( I \) state) is longer than 4 days, a condition that is likely to hold for virtually any patient. Finally, the condition in (16) ensures that the range of RVIs acceptable to patients includes only finite values, so that patients never rely entirely on unscheduled visits, an assumption that is likely to be true for an average patient in any practice. Note that the condition (16) is derived from (6).

**Proposition 4.** Under the traditional mode, consider the setting described by (14)-(16). Then, the following results hold.

1) A capitation physician chooses the highest possible RVI and panel size.
2) A fee-for-service physician chooses the lowest possible RVI and panel size.

We relegate all proofs to the Appendix. Proposition 4 states that, under mild conditions, a fee-for-service physician is incentivized to use the lowest RVI value consistent with patient preferences, and, thus, the lowest panel size. On the other hand, a capitation physician prefers to take on as many patients as possible and to see them as infrequently as possible consistent with patient preferences. This result is consistent with conclusions in the health economics and medical literature about the general “direction of incentives” for fee-for-service and capitation physicians (Gosden et al. 2000).
4.2. Panel Size and RVI Decisions Under the Non-Physician Provider Mode

In practice, non-physician providers can be used in a number of ways. One of the most common approaches involves a physician supervising several NPPs. Under this approach, the routine care (e.g., history taking, physical examination) is provided by an NPP, while a physician gets involved only during a fraction of an appointment time. The use of NPPs can also take the form of delegating some of the appointments entirely to the NPPs, who have an option of consulting with the physician if needed. In all cases, using NPPs enables the physician to effectively increase her service capacity.

We characterize the optimal solution to the physician’s problem (12) in the presence of NPPs below. Note that, similar to Propositions 4, we consider the setting where patients do not rely exclusively on unscheduled visits. This is ensured if the cost parameter $c$ and the flexibility parameter $\Delta$ are such that

$$c(1 - \Delta) > \frac{(p_1 + \gamma p_2)^2}{p_1 p_2} - 1. \quad (17)$$

In addition, we focus on settings where the quality of the service provided by non-physician providers is sufficiently high, so that

$$\gamma \geq \left(1 + \frac{5}{3}\left(\frac{\sqrt{k + 1}}{k}\right)\right) \frac{\tau p_1}{p_2} - \frac{p_1}{p_2}, \quad (18)$$

where $k = \frac{4s}{\sqrt{5}}$.

**Proposition 5.** For the non-physician provider mode, consider a setting described by (15), (17), and (18). Then, the following results hold.

1) A capitation physician chooses the highest possible RVI and panel size.

2) If the time savings from using non-physician providers are low, so that

$$\tau n \geq \frac{k}{k + 1 + \sqrt{k + 1}}, \quad (19)$$

a fee-for-service physician chooses the lowest possible RVI and panel size. On the other hand, if the time savings from using non-physician providers are high, so that

$$\tau n \leq \frac{k}{k + 1 + \frac{3(c(1 + \Delta) + 1)}{2} + \sqrt{(1 + \frac{3(c(1 + \Delta) + 1)}{2})^2 + k(1 + 3(c(1 + \Delta) + 1))}}, \quad (20)$$

a fee-for-service physician chooses the highest possible RVI and panel size.

Proposition 5 indicates that using NPPs preserves the direction of incentives for a capitation physician as long as $p_1$ and $p_2$ are reasonably low, $c$ is reasonably high and (18) holds instead of (14). In other words, a capitation physician, under conditions of part 1) of Proposition 5, will choose the highest possible RVI value compatible with patients’ preferences. Note that (18), while
somewhat more restrictive than (14), is still likely to hold in any realistic setting. Consider, for
example, a full-time equivalent (FTE) physician with \( s = 24 \) daily appointment slots that would
like to ensure \( \delta = 0.975 \) (so that \( z_\delta = 1.96 \)). For such a physician, (18) holds for any expected
duration of office visit under NPPs, \( \tau^n \in [0, 1] \), and \( p_1 = p_2 \), as long as the quality parameter for
non-physician visit \( \gamma \) exceeds a fairly low threshold of 0.34. Also, if \( p_2 \neq p_1 \), this threshold peaks
at 0.45. It is also possible that (18) holds for any \( \gamma \), e.g., if \( p_2 = 0.5p_1 \), the threshold value becomes
-0.10. Even for a physician with \( s = 10 \) daily appointment slots, a threshold that \( \gamma \) has to exceed
for (18) to hold under \( z_\delta = 1.96 \) and \( p_1 = p_2 \) is still a low 0.54. In a similar manner, in fee-for-service
settings, the direction of physician incentives will remain unchanged as long as \( \tau^n \) is high enough
to satisfy (19), so that the physician involvement in the presence of NPPs remains comparable to
that under the traditional care mode. On the other hand, as indicated by part 2) of Proposition
5, a fee-for-service physician may switch from the lowest possible RVI to the highest possible one
as long as the physician time savings afforded by the use of NPPs are substantial.

We illustrate the intuition behind this result with the case where \( \tau^n = 0 \). Note that the capacity
constraint in (12) will be satisfied as an equality, so that, in this case, \( \hat{N}^n_{FFS} \hat{r}^n_{FFS} = s \). In other words,
the physician’s schedule is shielded from variability associated with unscheduled visits. Also, the
expected daily compensation for a fee-for-service physician is proportional to
\( \frac{\hat{N}^n_{FFS}}{\hat{r}^n_{FFS}} + \hat{N}^n_{FFS} \hat{\pi}_{U,FFS} = s + \hat{N}^n_{FFS} \hat{\pi}_{U,FFS} \), where both \( \hat{N}^n_{FFS} \) and \( \hat{\pi}_{U,FFS} \) are increasing functions of \( \hat{r}^n_{FFS} \)
(under (18)). Thus, the only variable component of physician’s expected daily compensation is provided by unscheduled
visits, and in order to maximize that contribution a physician will choose the highest possible RVI
value, i.e., \( \hat{r}^n_{FFS} = r^n_\Delta \).

### 4.3. Impact on Patient Health, Panel Size, and Physician Compensation

In this section, we use the results of Propositions 4 and 5 to compare patient health, panel size,
and the physician’s expected daily compensation under the NPPs with those under the traditional
care mode.

**PROPOSITION 6.** For the non-physician provider mode, consider a setting described by (15),
(16), and (18). Then, the following results hold.
1) For a capitation physician, patient health improves.
2) For a fee-for-service physician, patient health improves if time savings from using non-physician
providers are low or if patients accept a narrow range of RVIs. In addition, patient health decreases
if time savings from using non-physician providers are high, the quality of non-physician provider
service is high, and patients accept a wide range of RVIs.

As Proposition 3 states, patients respond to the use of NPPs by decreasing both the lower and
upper bounds of the acceptable range of RVI values. As a result, if the direction of physician
incentives does not change upon the introduction of NPPs, the optimal RVI value decreases as well. However, as part 2) of Proposition 6 indicates, a fee-for-service physician can opt for higher RVI value as compared to the traditional mode provided that she is managing a flexible patient panel and that the use of NPPs results in both high quality of service and low physician involvement.

Next, we focus on the effect of using NPPs on the average daily probability of patients initiating an unscheduled office visit, \( \hat{\pi}_{U,j} \). Two opposing effects are at work; on the one hand, as indicated by Proposition 6, the use of NPPs may decrease the optimal RVI value, thus decreasing the probability of an unscheduled visit. On the other hand, the quality of care during an unscheduled office visit may be lower if NPPs are used, leading to an increase in the frequency of those visits.

The other insight from Proposition 6 is that in settings where the introduction of NPPs does not affect the direction of physician incentives as compared to the traditional care mode, patient health, as measured by the probability of initiating an unscheduled office visit, improves. Note that if a physician is incentivized on a fee-for-service basis, and (18) and (20) hold, the health of patients belonging to a flexible panel will decrease despite a high-quality care provided using NPPs. Also, the addition of NPPs decreases the quality of care received by patients during the unscheduled office visits. Therefore, for any value of \( c \) in (5), the use of NPPs leads to lower utility for patients.

Finally, we describe the impact of using NPPs on patient panel size and the physician’s expected daily compensation.

PROPOSITION 7. For the non-physician provider mode, consider a setting described by (15), (16), and (18). Then, the following results hold.

1) For a capitation physician, expected daily compensation and patient panel size increase if the quality of non-physician provider service and the time savings from using non-physician providers are high.

2) For a fee-for-service physician, the expected daily compensation does not decrease. Also, patient panel size increases if the quality of non-physician provider service and the time savings from using non-physician providers are high, provided that (19) holds.

Use of non-physician providers is often associated with potential for increase in patient panel sizes since a portion of patient demand is handled by NPPs. Proposition 7 shows that, if patients adjust their preferences upon the introduction of NPP-based care, the optimal panel size will indeed increase in settings where both the quality of provided care and the time savings from using NPPs stay high. At the same time, if the use of NPPs results in a low quality of care while still requiring high level of physician involvement, the optimal panel size may decrease.

Another important implication of Proposition 7 (part 2) is that the expected daily compensation for a fee-for-service physician increases as long as (15) and (16) hold. Note that the addition
of NPPs increases a fee-for-service physician’s capacity to deal with unscheduled office visits by reducing physician’s involvement in those visits, so it is trivial that, ceteris paribus, a fee-for-service physician’s expected daily compensation is decreasing as a function of her involvement in unscheduled office visits, $\tau^n$. Thus, if $\tau^n = 1$, NPPs offer no increase in physician’s capacity to deal with unscheduled visits, so the case where $\tau^n = 1$ corresponds to the lowest level of expected daily compensation among all possible values of $\tau^n$. We show that the expected daily compensation for a fee-for-service physician increases compared to the traditional case even if $\tau^n = 1$. Note that, in this case, a fee-for-service physician’s expected daily compensation is given by 

$$s - \sqrt{\hat{N}_{FFS}^{n} \hat{\pi}_{U,FFS}^{n}},$$

(21)

is the standard deviation of the number of unscheduled office visits. Consider the case where a fee-for-service physician chooses the office revisit interval $r^n$. Note that this is a value of RVI which may not be optimal for her. Proposition 6 shows that $\hat{r}_{FFS}^{n}$ and $\hat{\pi}_{U,FFS}^{n}$ both decrease as compared to the traditional mode. Thus, the standard deviation of the number of unscheduled office visits, as shown in (21), decreases, and physician’s expected daily compensation increases.

In summary, we showed that a fee-for-service physician earns more revenue under the NPPs case even if she chooses a suboptimal RVI value. Recall that physician’s profit is a function of both expected daily compensation and cost of hiring NPPs, so an increase in physician’s expected daily compensation does not necessarily result in an increase in physician’s profit. For example, a fee-for-service physician’s profit increases only if the value of $\Pi_{FFS}^{n} - \Pi_{T}^{T}$ is greater than the fixed cost of hiring the NPP.

5. Numerical Examples

In this Section we provide numerical illustrations for the results of Propositions 4-7 using realistic values of problem parameters. We focus on the symmetric case for our numerical example, i.e., $p_1 = p_2 = p$, therefore the patient panels are characterized by three parameters: the daily health deterioration rate $p$, the cost associated with falling sick $c$, and the RVI flexibility fraction $\Delta$. Below we express several quantities characterizing patient demand for care in terms of these three parameters and estimate $p$, $c$ and $\Delta$ for a “typical” panel using estimates of patient demand for primary care available in the extant literature.

First, we would like to focus on the average rate of primary care office visits. The 2010 National Ambulatory Medical Care Survey (Centers for Disease Control and Prevention 2010) estimates the annual rate of primary care visits as 1.85 per patient. However, this number is obtained by dividing the total number of patient visits in a year by 303.7 million, the total population of the United
State. Since, according to United States Census Bureau (2010), the number of insured patients in the United States was estimated to be 256.2 million in 2010, we set the annual rate of primary care visits to $1.85 \times 303.7/256.2 = 2.19$ per patient to account for patient insurance status. This estimate translates into the daily rate of $R = 2.19/250$, where we have used 250 as an estimate for the number of working days in a year.

In our model, patient office visits are a combination of unscheduled visits and scheduled visits. For each patient, if the RVI is set at $r$, the average daily rate of scheduled visits is $\frac{1}{r}$, and the average daily rate of unscheduled visits under the traditional care mode is $\pi_U$. Also, note that a physician can be compensated on the fee-for-service or the capitation basis. For this numerical exercise, we assume that a fraction $\lambda$ of physicians are compensated on the fee-for-service basis, and the rest are compensated on the capitation basis. The value of this fraction can be estimated as follows. Using 2010 National Ambulatory Medical Care Survey we can estimate that the fraction of physicians revenue coming from the capitation payment method is around 6%. Note that physicians were asked to respond to the question regarding the payment method using 25%-spaced intervals, and we assign all observations within an interval to its midpoint to get this average estimate. Assuming that the rest of physician’s revenue comes from fee-for-service payments, we get $\lambda = 0.94$. Therefore, the average daily rate of office visits can be expressed as

$$R = \lambda \left( \frac{1}{r^+} + \pi_U^{1}(p, r^+) \right) + (1 - \lambda) \left( \frac{1}{r^+} + \pi_U^2(p, r^+) \right).$$ (22)

Further, around $f = 38.73\%$ of all primary care visits are related to “flare-ups” and “new conditions” (Centers for Disease Control and Prevention 2010), and we interpret such visits as being unscheduled. In our model, the average fraction of unscheduled visits under the traditional care mode can be expressed as

$$f = \frac{\lambda \left( \frac{\pi_U^{1}(p, r^-)}{r^+} \right) + (1 - \lambda) \left( \frac{\pi_U^2(p, r^-)}{r^+} \right)}{\lambda \left( \frac{\pi_U^1(p, r^+)}{r^+} \right) + (1 - \lambda) \left( \frac{\pi_U^2(p, r^+)}{r^+} \right)}.$$. (23)

Finally, the literature on physician incentives indicates that the number of patient visits changes significantly under different physician compensation schemes (see, for example, a review by Gosden et al. 2000). In one of the most recent studies on the effect of physician remuneration on the number of patient visits (Devlin and Sarma 2008), it is estimated that the number of patient visits increases by $C = 52\%$ when a physician moves from a “non-fee-for-service” compensation scheme to a fee-for-service scheme. The authors define “non-fee-for-service” physicians as ones who “obtain 90\% or more of their professional income from a non-fee-for-service scheme”. While “non-fee-for-service” scheme is not necessarily equivalent to a capitation scheme, we will use this value in our analysis to approximate the change in RVI resulting from a switch from fee-for-service to capitation
compensation. In our model, the relative change in the patient visit rate between fee-for-service and capitation settings under the traditional mode of care can be expressed as

\[
C = \frac{\left(\frac{1}{r^t} + \pi_U^t(p, r^t_+) \right) - \left(\frac{1}{r^t} + \pi_U^t(p, r^t_-) \right)}{\left(\frac{1}{r^t} + \pi_U^t(p, r^t_-) \right)}.
\]  

(24)

Using the estimates for \( R, f, C, \) and \( \lambda \), we use (22), (23), and (24) to obtain the estimates for \( p, c, \) and \( \Delta \) for a “typical” patient panel:

\[
p = 0.0081, \quad c = 3.79, \quad \Delta = 0.19.
\]  

(25)

Note that the corresponding values of \( r^t_- \) and \( r^t_+ \) for these parameter values are reasonable, standing at 157 and 410 days, respectively. Thus, a patient on a “typical” panel opts for, roughly, at least 5 months and at most 13 months between scheduled visits.

In order to compare the optimal decisions under alternative primary care modes, we also need to estimate physician capacity \( s \) and tolerance for overtime \( z_d \). In our analysis, we consider a full-time-equivalent physician who works 8 hours each day (Office of Management and Budget 2011). The average duration of office visits is assumed to be 20 minutes which is consistent with 22.1, 19.3, 18.7 and 18.4 minutes reported in 2010 National Ambulatory Medical Care Survey as the average durations of face-to-face interaction during an office visit for physicians in internal medicine, general and family practice, OB/GYN and pediatric fields, respectively. Therefore, we set \( s = 24 \). For the value of \( z_d \) we use 1.96 which corresponds to the overtime probability of 2.5% on each day. Based on these estimates, our “typical” panel size range is around 2,200 under fee-for-service and 3,000 under capitation incentives. These values are consistent with the average panel size of 2,303 obtained from a physician survey conducted by Alexander et al. (2005).

Next, we consider a “typical” panel described by (25) and illustrate how the changes in the key outcome values, the patient panel size \( \hat{N}^m_j \) and the patient health, as described by the probability of generating an unscheduled visit \( \hat{\pi}_U^n, j = FFS, CAP \), are influenced by the service parameters \( \tau^n \) and \( \gamma \) under non-physician providers.

Under non-physician providers, Proposition 6 establishes that, under mild conditions, the patient health improves as compared to the traditional case for a capitation physician. For a fee-for-service case, Figure 7 illustrates a similar comparison between the NPP and the traditional modes. In particular, patient health improves in settings where physician’s involvement in each unscheduled visit is comparable to that under the traditional care mode. Note that the value of \( \tau^n \) for \( \hat{\pi}_U^n,FFS = \hat{\pi}_U^n,FFS \) is not sensitive to the quality of care provided using NPPs.

Figure 8 shows how the use of non-physician providers affects the patient panel size. The key factors determining whether a physician will choose larger or smaller panel size as compared to the
As expected, if the quality of care provided by the NPPs is sufficiently high and the fraction of time that a physician spends on an unscheduled visit is sufficiently low, the panel size increases as compared to the traditional case under both fee-for-service and capitation incentives. The “kinks” on the curve $\hat{N}_{FFS}^n$ in Figure 8(a) highlight the changes in the way that a fee-for-service physician assigns RVI as the quality of care decreases. In this figure, point $A$ ($\gamma = 1, \tau^n = 1$) represents the traditional care mode and, for settings between points $A$ and $B$, a fee-for-service physician chooses the lowest possible RVI value (i.e., $\hat{r}_{FFS}^n = r^n_-$). Between points $B$ and $C$, the fee-for-service physician chooses “interior” RVI values ($r_-^n \leq \hat{r}_{FFS}^n \leq r_+^n$), while between points $C$ and $D$ the highest possible RVI value is chosen ($\hat{r}_{FFS}^n = r_+^n$). Note that there are no kinks in Figure 8(b) since, in this case, a capitation physician always chooses the highest possible RVI value.

We conclude this section by exploring patient heterogeneity. Note that 2.19 yearly visits per
patient is an average value over all patients on the panel. This average value masks the variability that exists in patient panels. Ideally, we might examine patient heterogeneity by chronic condition or other ailments, but unfortunately, the National Ambulatory Care Survey does not provide visit rates by disease category. Instead, we have this information by demographic categories, so we focus on those aged 65+ as this group is likely to have a higher prevalence of chronic conditions. Among this sample, we calculate that the average number of yearly office visits is 4.38, which is double the estimate for the pooled patient group (2.19). We obtained this estimate using the following method. The annual rate of office visits for patients who are 65 and above is 6.66 visits per year (primary care and non-primary care). We also know that 55.5% of all patient visits are primary care visits. Assuming that this ratio is constant for all ages, and multiplying by the insurance factor we obtain $6.66 \times 0.555 \times 303.7/256.2 = 4.38$.

This doubling in yearly office visits for patients aged 65+ suggests that if we were to conduct the entire numerical analysis on this subgroup, we would estimate significantly smaller RVI values. Not surprisingly, this is what we find: we re-estimated our structural model based on the 4.38 yearly visits (and all other parameters held constant), and find that the corresponding range of acceptable RVI values is 65 to 120 days (as opposed to 157 to 410 days for the pooled population). In particular, we use the structural model that was introduced in (22), (23), and (24) for a panel that consists of only patients aged 65 and above, i.e., $R = 4.38/250$. The estimates for $p$, $c$, and $\Delta$ for such a patient panel are the following:

$$p = 0.011, \quad c = 6.66, \quad \Delta = 0.33. \quad (26)$$

For a patient panel with only 65+ patients, we observe higher values of $p$ and $c$. This is consistent with higher sickness rates in this population and their higher risks associated with being sick.

6. Conclusion

The primary care system in the United States is facing challenging times owing to factors such as the ageing population and physician shortage. To accommodate this mismatch between supply and demand, the primary care system may have to augment the traditional care delivery mode with other approaches such as using non-physician providers. Both patients and physicians may actively respond to these alternatives, impacting important system outcomes such as patient panel size and office revisit intervals. Understanding these changes is crucial for designing effective policies to aid a safe transition in primary care services.

Our study addresses the complexity of physician and patient interactions in response to non-physician providers. In our model, patients respond to changes in the way care is delivered by adjusting the range of office revisit interval values they are willing to accept. On the physician’s
side, we consider fee-for-service and capitation compensation schemes, and model the physician’s choice of patient panel size and office revisit intervals consistent with patient preferences. For each care delivery mode, we characterize the optimal RVI and panel size values as well as the resulting patient health levels and the physician’s expected daily compensation. We use the realistic estimates of problem parameters to illustrate the resulting outcomes for a “typical” patient panel and a “typical” physician for a wide range of values characterizing the care quality and the time savings associated with alternative care delivery modes.

We show that considering patient and physician responses to the changes in primary care delivery modes (e.g., non-physician providers) is important since, under many plausible scenarios, these responses influence the magnitude and even the direction of changes in system outcomes. Under non-physician providers, if patients do not respond to the potential decrease in the quality of care, their overall health can decline. Yet, as our results show, patient health may often improve if patient response is accounted for. In addition, under non-physician providers, the presence of active patient response may limit the increase in patient panel sizes.

This paper provides the first attempt at modeling both the patient and physician responses to the care delivery innovations such as the use of non-physician providers in primary care. Our analysis aims at providing practice-oriented recommendations that are based on a limited number of easy-to-estimate parameters. As a result, it relies on several simplifying assumptions.

First, we model patient panels as being homogeneous. In reality, patients may display a significant degree of heterogeneity with respect to many observable and unobservable characteristics, and, as a result, any patient panel will include sub-populations with substantially different levels of health decline rates, disutilities associated with being sick, and level of awareness of their health dynamics. Consequently, it may be necessary to recast the issue of setting the “right” panel size as a problem of finding the optimal “portfolio” of different patient groups. While such an approach may allow for extraction of additional efficiency, it also raises a set of potential ethical and legal issues associated with limiting the ability for specific groups of patients to join the physician’s panel. We believe, however, that a simplified approach we employ goes a long way in guiding the panel-sizing decisions. In addition, our model has a single patient health parameter (the average value across all patients in the panel), which ignores heterogeneity in patient health. Analytically, this simplification enabled us to obtain closed-form expressions for the RVI values and panel size. We hope that our analysis provides a first step for future model extensions that explore the important role of patient heterogeneity in our setting.

Second, we assume that physician’s appointment capacity consists of a given number of one-size-fits-all appointments. In practice, new patient visits, follow-up visits, and annual check-up visits may have different impacts on physician’s service capacity. As a result, some of those visits may be
scheduled for two appointment slots instead of one. Since we assume that each patient visit takes a single appointment slot, the panel size values we derive present upper bounds on the values that can be achieved in settings where each visit, on average, requires higher level of service capacity.

Third, we assume that a physician uses an “open access” approach to handling patient visits. In particular, in our model a patient that falls sick gets the same-day access to treatment, and there is no backlog of patient appointments. This assumption allows for closed-form characterization of patient health dynamics under alternative modes of care delivery. In our setting, the presence of an appointment backlog for routine appointments will most likely lead to fluctuations in realized frequency of scheduled appointments. On the other hand, existence of backlog for unscheduled appointments implicitly raises the cost of an unscheduled visit for the patients because a “sick” patient either has to visit the emergency room before coming to the physician or remain sick while waiting for care. Overall, while “open access” has gained a wider acceptance in recent years, appointment backlogs are common in practice, and a potential extension to our work would include that feature.

Fourth, in our model we assume that physician compensation is the same for unscheduled visits in the presence of non-physician providers and in the traditional case. This is consistent with the “incident to” billing mechanism which allows a nurse practitioner to bill for services under a physician’s National Provider Identification number (NPI) at 100% reimbursement rate (Weiland 2008). The analysis of more complicated remuneration schemes is one potential direction for future research. Also, we focused our analysis on assessing the impacts of NPPs on patient health and physician compensation. Analyzing the addition of NPPs from a social planner perspective with outcomes such as social welfare or surplus are promising directions for future research that are unfortunately beyond the scope of the present analysis.

Finally, our analysis is focused on two main goals. On the one hand, we aim to provide guidance to practitioners on the issues of panel sizing and the frequency of scheduled office visits based on a model with small number of easy-to-estimate parameters. While the prescriptions we derive using our parsimonious model are necessarily coarse, we believe they offer a useful practical alternative when compared to detailed and more precise, but also more costly, simulation-based analysis. On the other hand, our analysis enables us to derive important policy-related insights about the impact of non-physician providers on key primary care performance metrics such as patient health, physician compensation, and patient coverage.

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Bavafa, Savin, Terwiesch: Managing Patient Panels


Appendix for “Managing Patient Panels with Non-Physician Providers”

Proof of Proposition 1

When the office revisit interval value is set to $r$, a patient visits his physician every $r$ days, and upon such visit, patient’s health is restored to the state $H$. Consider a “cycle” of $r$ days, starting at $\tau = 0$ and ending at $\tau = r$, and let $Q^H_\tau$, $Q^I_\tau$ and $Q^S_\tau$ denote the probabilities for a patient to be in the state $H$, $I$ and $S$ at time $\tau$, respectively. Based on $\frac{dQ_\tau}{d\tau} = Q_\tau P$ we can write the following evolution equations:

\[
\frac{dQ^H_\tau}{d\tau} = -p_1 Q^H_\tau + \omega Q^S_\tau, \\
\frac{dQ^I_\tau}{d\tau} = p_1 Q^H_\tau - p_2 Q^I_\tau, \\
\frac{dQ^S_\tau}{d\tau} = -\omega Q^S_\tau + p_2 Q^I_\tau,
\]

with

\[Q^H_\tau + Q^I_\tau + Q^S_\tau = 1, \tag{A4}\]

and the initial conditions

\[
\begin{bmatrix}
Q^H_0 \\
Q^I_0 \\
Q^S_0
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}, \tag{A5}
\]

since the patient starts each cycle in the state $H$.

Introducing the Laplace transforms

\[Q^i(s) = \int_0^\infty d\tau e^{-\tau s} Q^i_\tau, i = H, I, S, \tag{A6}\]

and applying them to (A1)-(A5) we get

\[-1 + sQ^H(s) = -p_1 Q^H(s) + \omega Q^S(s), \tag{A7}\]
\[sQ^I(s) = p_1 Q^H(s) - p_2 Q^I(s), \tag{A8}\]
\[sQ^S(s) = -\omega Q^S(s) + p_2 Q^I(s), \tag{A9}\]

so that

\[Q^I(s) = \frac{p_1}{(p_1 + s)(p_2 + s) - \frac{\omega}{\omega + s} p_1 p_2}. \tag{A10}\]

In the limit of $\omega \to \infty$,

\[Q^I(s) = \frac{p_1}{s(p_1 + p_2 + s)} = \frac{p_1}{p_1 + p_2} \left( \frac{1}{s} - \frac{1}{s + p_1 + p_2} \right), \tag{A11}\]

so that, applying the inverse Laplace transform, we get

\[Q^I_\tau = \frac{p_1}{p_1 + p_2} \left( 1 - e^{-(p_1 + p_2)\tau} \right). \tag{A12}\]
The average time spent in the state $I$ over an $r$-day cycle is

$$Q_I = \frac{1}{r} \int_0^r \frac{p_1}{p_1 + p_2} (1 - e^{-(p_1 + p_2)r}) \, d\tau = \frac{p_1}{p_1 + p_2} \left( 1 - \frac{1 - e^{-(p_1 + p_2)r}}{(p_1 + p_2)r} \right),$$

(A13)

and

$$\pi_I^r(p_1, p_2, r) = \frac{p_1 p_2}{p_1 + p_2} \left( 1 - \frac{1 - e^{-(p_1 + p_2)r}}{(p_1 + p_2)r} \right).$$

(A14)

**Proof of Proposition 2**

Using $\frac{dQ}{d\tau} = Q \cdot P^n$ we can write the following evolution equations:

$$\frac{dQ_H}{d\tau} = -p_1 Q_H + \omega \gamma Q_S,$$

(A15)

$$\frac{dQ_I}{d\tau} = p_1 Q_H - p_2 Q_I + \omega(1 - \gamma) Q_S,$$

(A16)

$$\frac{dQ_S}{d\tau} = -\omega Q_S + p_2 Q_I,$$

(A17)

with

$$Q_H + Q_I + Q_S = 1,$$

(A18)

and initial conditions

$$\begin{bmatrix} Q_H^0 \\ Q_I^0 \\ Q_S^0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

(A19)

since the patient starts each cycle in the state $H$.

Introducing the Laplace transforms

$$Q^i(s) = \int_0^\infty d\tau e^{-s \tau} Q^i_\tau, \quad i = H, I, S,$$

(A20)

and applying them to (A15)-(A19) we get

$$-1 + s Q^H(s) = -p_1 Q^H(s) + \omega \gamma Q^S(s),$$

(A21)

$$s Q^I(s) = p_1 Q^H(s) - p_2 Q^I(s) + \omega(1 - \gamma) Q^S(s),$$

(A22)

$$s Q^S(s) = -\omega Q^S(s) + p_2 Q^I(s),$$

(A23)

so that

$$Q^I(s) = \frac{p_1}{(p_1 + s)(p_2 + s) - \frac{p_1}{\omega \gamma} (p_1 p_2 + (1 - \gamma)p_2 s)}.$$

(A24)

In the limit of $\omega \to \infty$,

$$Q^I(s) = \frac{p_1}{s(p_1 + \gamma p_2 + s)} = \frac{p_1}{p_1 + \gamma p_2} \left( \frac{1}{s} - \frac{1}{s + p_1 + \gamma p_2} \right),$$

(A25)
so that, applying the inverse Laplace transform, we get

\[ Q^* = \frac{p_1}{p_1 + \gamma p_2} \left( 1 - e^{-(p_1 + \gamma p_2)r} \right). \]  

(A26)

The average time spent in the state \( I \) over an \( r \)-day cycle is

\[ \bar{Q}_I = \frac{1}{r} \int_0^r \frac{p_1}{p_1 + \gamma p_2} \left( 1 - e^{-(p_1 + \gamma p_2)r} \right) d\tau = \frac{p_1}{p_1 + \gamma p_2} \left( 1 - \frac{1 - e^{-(p_1 + \gamma p_2)r}}{(p_1 + \gamma p_2)r} \right), \]  

(A27)

and

\[ \pi^U (p_1, p_2, r) = p \bar{Q}_I = \frac{p_1 p_2}{p_1 + \gamma p_2} \left( 1 - \frac{1 - e^{-(p_1 + \gamma p_2)r}}{(p_1 + \gamma p_2)r} \right). \]  

(A28)

**Proof of Proposition 3**

For the traditional mode of care, the unconstrained minimum of (5) \( r^t \) satisfies

\[ -\frac{1}{(r^t)^2} + \frac{(1 + c)p_1 p_2}{(p_1 + p_2)^2 (r^t)^2} \left( 1 - ((p_1 + p_2)r^t + 1)e^{-(p_1 + p_2)r^t} \right) = 0, \]  

(A29)

or

\[ Ze^Z = -\frac{(1 + c)p_1 p_2 - (p_1 + p_2)^2}{(1 + c)p_1 p_2 e}, \]  

(A30)

where \( Z = (p_1 + p_2)r^t - 1 \). Thus,

\[ r^t_0 = -\frac{1}{p_1 + p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + p_2)^2}{(1 + c)p_1 p_2} - 1 \right) \right) \right), \]  

(A31)

where \( W_{-1} \) is the lower branch of the Lambert W function (Corless et al. 1996), defined on the interval \([-\frac{1}{e}, 0]\) and taking values in the range from \(-1\) to \(-\infty\). Note that (A29) can change sign at most once, and the limits of the expression in (A29) for \( r^t \to 0 \) and \( r^t \to \infty \) are \(-\infty\) and 0, respectively. Thus, for \( c \leq \frac{(p_1 + p_2)^2}{p_1 p_2} - 1 \), the patient cost is minimized when \( r^t \to \infty \). In a similar fashion, under non-physician providers we get

\[ -\frac{1}{(r^n)^2} + \frac{(1 + c)p_1 p_2}{(p_1 + \gamma p_2)^2 (r^n)^2} \left( 1 - ((p_1 + \gamma p_2)r^n + 1)e^{-(p_1 + \gamma p_2)r^n} \right) = 0, \]  

(A32)

\[ r^n_0 = -\frac{1}{p_1 + \gamma p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + \gamma p_2)^2}{(1 + c)p_1 p_2} - 1 \right) \right) \right), \]  

(A33)

Also, for \( c \leq \frac{(p_1 + \gamma p_2)^2}{p_1 p_2} - 1 \), the patient cost is minimized when \( r^n \to \infty \).
Proof of Corollary 1

Part a) can be proved in the following way:

\[ r^t_{o,A} = -\frac{1}{\alpha + \beta} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{\alpha + \beta}{(1+c)\alpha \beta} - 1 \right) \right) \right) \]
\[ = -\frac{1}{\beta + \alpha} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{\beta + \alpha}{(1+c)\beta \alpha} - 1 \right) \right) \right) = r^t_{o,B}. \] (A34)

For part b), first consider the case in which \( \alpha \leq \beta \). In such a case, \( \alpha + \gamma \beta \leq \beta + \gamma \alpha \) because \( \gamma \leq 1 \).

The expression for \( r^o_{n,A} \) can be written as

\[ r^o_{n,A} = -\frac{1}{\alpha + \gamma \beta} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{\alpha + \gamma \beta}{(1+c)\alpha \beta} - 1 \right) \right) \right) \]
\[ = -\frac{1}{x \sqrt{e(1+c)\alpha \beta}} \left( 1 + W_{-1} \left( x^2 - \frac{1}{e} \right) \right), \] (A35)

where

\[ x = \frac{\alpha + \gamma \beta}{\sqrt{e(1+c)\alpha \beta}}. \] (A36)

Note that swapping the values of \( \alpha \) and \( \beta \) in (A36) increases the value of \( x \) as it increases the numerator and leaves the denominator unchanged:

\[ x \leq x' = \frac{\beta + \gamma \alpha}{\sqrt{e(1+c)\alpha \beta}}. \] (A37)

Also, the function \( g(x) = -\frac{1}{x} \left( 1 + W_{-1} \left( x^2 - \frac{1}{e} \right) \right) \) is an increasing function of \( x \) for all feasible values of \( x \). Thus, using (A35) and (A37):

\[ r^o_{n,A} = -\frac{1}{x' \sqrt{e(1+c)\alpha \beta}} \left( 1 + W_{-1} \left( \left( x' \right)^2 - \frac{1}{e} \right) \right) \leq -\frac{1}{x' \sqrt{e(1+c)\alpha \beta}} \left( 1 + W_{-1} \left( \left( x' \right)^2 - \frac{1}{e} \right) \right) = r^o_{n,B}. \] (A38)

The proof is similar for the case in which \( \alpha > \beta \).

Proof of Proposition 4

Under the traditional approach to care delivery, consider a physician with the daily number of appointments \( s \) satisfying (14), and a patient panel characterized by the daily health decline rates \( p_1 \) and \( p_2 \) satisfying (15), the cost parameter \( c \) and the flexibility parameter \( \Delta \) such that (16) holds. Then, let \( \hat{\pi}^t_{U,j} \) be expressed by (13) and \( W_{-1} \) designate the lower branch of the Lambert W function. For part 1) of Proposition 4, we would like to show that, for a capitation physician, the optimal RVI and panel size values are given by

\[ \hat{r}_{CAP}^t = -\frac{1}{p_1 + p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + p_2)^2}{(1 + c(1 - \Delta))p_1 p_2} - 1 \right) \right) \right), \] (A39)
\[ \hat{N}_{t_{CAP}} = \left( \frac{2s}{z_\delta \left( \bar{\pi}^t_{U_{CAP}} + \frac{4s}{z_\delta} \left( \bar{r}^t_{CAP} + \hat{\pi}^t_{U_{CAP}} \right) + \sqrt{\pi^t_{U_{CAP}}} \right) \right)^2. \] \] (A40)

For part 2) of Proposition 4, we will establish that, for a fee-for-service physician, the optimal RVI and panel size values are given by

\[ \hat{r}^t_{FFS} = -\frac{1}{p_1 + p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + p_2)^2}{(1 + c(1 + \Delta))p_1p_2} - 1 \right) \right) \right), \] \] (A41)

\[ \hat{N}^t_{FFS} = \left( \frac{2s}{z_\delta \left( \bar{\pi}^t_{U_{FFS}} + \frac{4s}{z_\delta} \left( \bar{r}^t_{FFS} + \hat{\pi}^t_{U_{FFS}} \right) + \sqrt{\pi^t_{U_{FFS}}} \right) \right)^2. \] \] (A42)

Note that, irrespective of the physician’s compensation scheme, both the objective function and the left-hand-side of the capacity constraint in (12) are increasing functions of the patient panel size \( N \) for a given choice of \( r \). Thus, the optimal solution to (12) produces tight capacity constraint:

\[ \frac{\hat{N}^t_{FFS}}{\hat{r}^t_{FFS}} + \hat{\pi}^t_{U_{FFS}} + z_\delta \sqrt{\hat{N}^t_{FFS} \hat{\pi}^t_{U_{FFS}}} = s, \quad j = FFS, CAP, \] \] (A43)

or

\[ \hat{N}^t_{j} = \left( \frac{\sqrt{\bar{\pi}^t_{U_{j}} + \frac{4s}{z_\delta} \left( \frac{1}{\hat{r}^t_{j}} + \hat{\pi}^t_{U_{j}} \right) - \sqrt{\pi^t_{U_{j}}} \right)}{2} \right)^2, \quad j = FFS, CAP, \] \] (A44)

where we have denoted \( \hat{\pi}^t_{U_{j}} = \pi^t_{U}(p_1, p_2, \hat{r}^t_{j}) \) as defined in (13). To simplify this expression further, we multiply numerator and denominator of (A44) by \( \left( \sqrt{\bar{\pi}^t_{U_{j}} + \frac{4s}{z_\delta} \left( \frac{1}{\hat{r}^t_{j}} + \hat{\pi}^t_{U_{j}} \right) + \sqrt{\pi^t_{U_{j}}} \right)^2 \):

\[ \hat{N}^t_{j} = \left( \frac{2s}{z_\delta \left( \sqrt{\bar{\pi}^t_{U_{j}} + \frac{4s}{z_\delta} \left( \frac{1}{\hat{r}^t_{j}} + \hat{\pi}^t_{U_{j}} \right) + \sqrt{\pi^t_{U_{j}}} \right) \right)^2. \] \] (A45)

For part 1) of the Proposition, we want to show that \( \frac{d\hat{N}^t_{j}}{d\hat{r}^t_{j}} \geq 0 \). For this, it suffices to show that:

\[ \frac{d}{d\hat{r}^t_{j}} \left( \sqrt{\bar{\pi}^t_{U_{j}} + \frac{4s}{z_\delta} \left( \frac{1}{\hat{r}^t_{j}} + \hat{\pi}^t_{U_{j}} \right) + \sqrt{\pi^t_{U_{j}}} \right) \leq 0. \] \] (A46)

Note that according to (2)

\[ \frac{d\hat{\pi}^t_{U_{j}}}{d\hat{r}^t_{j}} = \frac{p_1^2}{\left( p_1 + p_2 \right)^2 \left( \hat{r}^t_{j} \right)^2} \left( 1 - \left( (p_1 + p_2)\hat{r}^t_{j} + 1 \right) e^{-(p_1 + p_2)\hat{r}^t_{j}} \right), \] \] (A47)
so that
\[ 0 \leq \frac{d\pi_{t',j}}{dt'} \leq \frac{A'}{\left(\tilde{r}_j^t\right)^2}, \tag{A48} \]
where \( A' = \frac{p_1 p_2}{(p_1 + p_2)^2} \). Using \( k = \frac{4x}{\pi} \), we can express the derivative on the left-hand side of (A46) as

\[
\begin{align*}
\frac{d\pi_{t',j}}{dt'} &\leq \frac{k \left( -\frac{1}{(r_j')^2} + \frac{A'}{(r_j')^2} \right)}{2\sqrt{\pi_{t',j} + k(\frac{1}{r_j'} + \tilde{\pi}_{t',j})}} + \frac{\frac{ds_{t',j}}{dt'}}{\sqrt{\pi_{t',j}}} \\
&\leq \frac{k \left( \frac{A'-1}{(r_j')^2} \right)}{2\sqrt{\pi_{t',j} + k(\frac{1}{r_j'} + \tilde{\pi}_{t',j})}} + \frac{\frac{ds_{t',j}}{dt'}}{\sqrt{\pi_{t',j}}} \\
&\leq \frac{1}{2\sqrt{\pi_{t',j} + \frac{1}{r_j'}}} \left( \frac{A'-1}{(r_j')^2} \right) \left( \frac{k}{\sqrt{k+1}} \right) + \frac{1}{2\sqrt{\frac{1}{\pi_{t',j} r_j'^2}}} \frac{d\pi_{t',j}}{dt'} \\
&\leq \frac{1}{2\sqrt{\pi_{t',j} + \frac{1}{r_j'}}} \left( -\frac{3}{4} \right) \left( \frac{k}{\sqrt{k+1}} \right) + \frac{1}{2\sqrt{\frac{1}{\pi_{t',j} r_j'^2}}} \frac{d\pi_{t',j}}{dt'}, \tag{A49}
\end{align*}
\]

where we used
\[ \max_{p_1, p_2 \geq 0} A' = \max_{p_1, p_2 \geq 0} \frac{p_1 p_2}{(p_1 + p_2)^2} = \frac{1}{4}. \tag{A51} \]

Note that
\[
\begin{align*}
\sqrt{1 + \frac{1}{\pi_{t',j} r_j'^2} \frac{d\pi_{t',j}}{dt'}} &= \frac{A'}{(r_j')^2} \sqrt{1 + \frac{1}{A'} \frac{1}{x - 1 + e^{-x}} \left( 1 - (1 + x)e^{-x} \right)} \\
&= \frac{\sqrt{A'}}{(r_j')^2} \sqrt{A' + \frac{1}{x - 1 + e^{-x}} \left( 1 - (1 + x)e^{-x} \right)} \\
&\leq \frac{1}{2(r_j')^2} \sqrt{\frac{4}{x - 1 + e^{-x}} \left( 1 - (1 + x)e^{-x} \right)} \\
&= \frac{1}{4(r_j')^2} \sqrt{\frac{4}{x - 1 + e^{-x}} \left( 1 - (1 + x)e^{-x} \right)}, \tag{A52}
\end{align*}
\]

where \( x = (p_1 + p_2)r_j'^t \). We observe that
\[ g(x) = \sqrt{\frac{1 + \frac{4}{x - 1 + e^{-x}} \left( 1 - (1 + x)e^{-x} \right)}{x - 1 + e^{-x} \left( 1 - (1 + x)e^{-x} \right)}}, \tag{A53} \]
is a unimodal function on \( x \geq 0 \), reaching the maximum of approximately 1.39 at \( x = 3.6 \). Thus, \( g(x) \leq \frac{3}{2} \) for any non-negative \( x \), and the last expression in (A50) can be bound as follows

\[
\frac{1}{2} \frac{1}{\sqrt{\hat{\pi}_{U,j} + \frac{1}{r_j}}} \left( -\frac{3}{4} \frac{k}{(\hat{r}_j^n)^2} \sqrt{k+1} + 2 \sqrt{1 + \frac{1}{\hat{\pi}_{U,j} \hat{r}_j^n} \frac{d\hat{\pi}_{U,j}}{d\hat{r}_j^n}} \right) \leq \frac{1}{2} \frac{1}{\sqrt{\hat{\pi}_{U,j} + \frac{1}{r_j}}} \left( -\frac{3}{4} \frac{k}{(\hat{r}_j^n)^2} \sqrt{k+1} + 3 \frac{3}{4} \frac{3}{(\hat{r}_j^n)^2} \right). \tag{A54}
\]

The expression inside the brackets in the last line above is non-positive for all \( k = \frac{4s}{z} \geq \frac{1 + \sqrt{5}}{2} \approx 1.6 \), and, in particular for all \( s \) values satisfying (14).

This proves that \( \hat{N}_j^n \) is a non-decreasing function of \( \hat{r}_j^n \), and, as a consequence, that a capitation physician’s compensation is an increasing function \( \hat{r}_C^n \). Therefore, choosing the revisit interval \( r_j^n \) is optimal for her.

For part 2) of the Proposition, we re-express the objective of a fee-for-service physician using (A43):

\[
\hat{\Pi}_t^{FFS} = s - z\delta \sqrt{\hat{N}_t^{FFS} \hat{\pi}_{U,FFS}}. \tag{A55}
\]

Since \( \frac{dn_{t,FFS}}{d\hat{r}_t^{FFS}} \geq 0 \) and \( \frac{dn_{U,FFS}}{d\hat{r}_t^{FFS}} \geq 0 \), the right-hand side of equation (A55) is a decreasing function of \( \hat{r}_t^{FFS} \). Thus, the optimal revisit interval for a fee-for-service physician is \( r_t^{FFS} \).

**Proof of Proposition 5**

Under the non-physician provider mode, consider a physician with the daily number of appointments \( s \) and a patient panel characterized by the daily health decline rate \( p \) satisfying (15), the cost parameter \( c \) and the flexibility parameter \( \Delta \) such that (17) holds. Let \( k = \frac{4s}{z} \). Also, let \( \hat{\pi}_{U,j}^n \) be expressed by (13) and \( W_{-1} \) designate the lower branch of the Lambert \( W \) function. For part 1) of Proposition 5, we would like to show that, for a capitation physician, the optimal RVI and panel size values are given by

\[
\hat{r}_{t,CAP} = -\frac{1}{p_1 + \gamma p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + \gamma p_2)^2}{(1 + c(1 - \Delta))p_1 p_2} - 1 \right) \right) \right), \tag{A56}
\]

as long as (18) holds, and

\[
\hat{N}_{t,CAP} = \left( \frac{2s}{z\delta \left( \sqrt{\hat{\pi}_{U,CAP} + \frac{4s}{z} \left( \hat{\pi}_{C,CAP} \hat{r}_{C,CAP} + \hat{\pi}_{U,CAP} \right) + \sqrt{\hat{\pi}_{U,CAP}^2}} \right)} \right)^2. \tag{A57}
\]

For part 2) of Proposition 5, we will establish that, for a fee-for-service physician, the optimal RVI and panel size values are given by

\[
\hat{r}_{t,FFS} = -\frac{1}{p_1 + \gamma p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + \gamma p_2)^2}{(1 + c(1 + \Delta))p_1 p_2} - 1 \right) \right) \right), \tag{A58}
\]
as long as both (18) and (19) hold, while
\[
\hat{r}_{FFS}^n = -\frac{1}{p_1 + \gamma p_2} \left( 1 + W_{-1} \left( \frac{1}{e} \left( \frac{(p_1 + \gamma p_2)^2}{1 + c(1 - \Delta)} p_1 p_2 - 1 \right) \right) \right), \quad (A59)
\]
as long as both (18) and (20) hold, and
\[
\hat{N}_{FFS}^n = \left( \frac{2s}{z_\delta \left( \sqrt{\hat{\pi}_{U,FFS}^n} + \frac{4s}{z_\delta} \left( \frac{1}{r_{FFS}} + \hat{\pi}_{U,FFS}^n \right) + \sqrt{\hat{\pi}_{U,FFS}^n} \right) \right)^2. \quad (A60)
\]

For capitation as well as fee-for-service physicians, both the objective function and the left-hand-side of the capacity constraint in (12) are increasing functions of the patient panel size \(N\) for a given choice of \(r\). Thus, the optimal solution to (12) produces binding capacity constraint:
\[
\frac{\hat{N}_{j}^n}{r_{j}^n} + \hat{N}_{j}^n \hat{\pi}_{U,j}^n - z_\delta \sqrt{\hat{N}_{j}^n \hat{\pi}_{U,j}^n} = s, \quad j = FFS,CAP, \quad (A61)
\]
or
\[
\hat{N}_{j}^n = \left( \sqrt{\hat{\pi}_{U,j}^n} + \frac{4s}{z_\delta} \left( \frac{1}{r_{j}^n} + \hat{\pi}_{U,j}^n \right) - \sqrt{\hat{\pi}_{U,j}^n} \right)^2, \quad j = FFS,CAP. \quad (A62)
\]

To simplify this expression further, we multiply numerator and denominator of (A62) by \(\left( \sqrt{\hat{\pi}_{U,j}^n} + \frac{4s}{z_\delta} \left( \frac{1}{r_{j}^n} + \hat{\pi}_{U,j}^n \right) + \sqrt{\hat{\pi}_{U,j}^n} \right)^2\):
\[
\hat{N}_{j}^n = \left( \frac{2s}{z_\delta \left( \sqrt{\hat{\pi}_{U,j}^n} + \frac{4s}{z_\delta} \left( \frac{1}{r_{j}^n} + \hat{\pi}_{U,j}^n \right) + \sqrt{\hat{\pi}_{U,j}^n} \right) \right)^2. \quad (A63)
\]

For the result in part 1) of the Proposition, we will show that under (18), \(d\hat{N}_{j}^n / d\hat{r}_{j}^n \geq 0\), or, equivalently, that
\[
\frac{d}{d\hat{r}_{j}^n} \left( \sqrt{\hat{\pi}_{U,j}^n} + \frac{4s}{z_\delta} \left( \frac{1}{r_{j}^n} + \hat{\pi}_{U,j}^n \right) + \sqrt{\hat{\pi}_{U,j}^n} \right) \leq 0. \quad (A64)
\]

Note that
\[
\hat{\pi}_{U,j}^n = \tau^n \pi_{U,j}^n. \quad (A65)
\]

Then, according to (4)
\[
\frac{\partial \hat{\pi}_{U,j}^n}{\partial \hat{r}_{j}^n} = \frac{p_1 p_2}{(p_1 + \gamma p_2)^2} \left( 1 - ((p_1 + \gamma p_2)\hat{r}_{j}^n + 1)e^{-(p_1 + \gamma p_2)\hat{r}_{j}^n} \right), \quad (A66)
\]
so that
\[
0 \leq \frac{\partial \hat{\pi}_{U,j}^n}{\partial \hat{r}_{j}^n} \leq \frac{A^n}{\left( \hat{r}_{j}^n \right)^2}, \quad (A67)
\]
where \( A^n = \frac{p_1p_2}{(p_1 + \gamma p_2)^2} \).

Note that we consider the case where \( \tau^n A^n \leq 1 \). This requires

\[
\gamma \geq \sqrt{\tau^n \left( \frac{p_1}{p_2} \right) \frac{p_1}{p_2}}. \tag{A68}
\]

Using \( k = \frac{4 \pi z}{(zk)^2} \), we can express the derivative on the left-hand side of (A64) as

\[
\frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}} + k \left( \frac{1}{(\tau^n)^2} + \frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}} \right) + \frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}} \leq \frac{k \left( \frac{A^n - \check{\tau}^n}{(\tau^n)^2} \right) \left( \frac{1}{(\tau^n)^2} \right) + \frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}}}{2 \sqrt{\hat{\nu}_{n,j} + \frac{\hat{\nu}_{n,j}}{\hat{\nu}_{n,j}}} + \frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}}} \leq \frac{1}{2 \sqrt{\hat{\nu}_{n,j} + \frac{\hat{\nu}_{n,j}}{\hat{\nu}_{n,j}}} + \frac{1}{\hat{\nu}_{n,j}}} \left( \frac{A^n \check{\tau}^n - 1}{(\check{\tau}^n)^2} \right) - \frac{k}{\sqrt{k+1}} \frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}} + 2 \sqrt{1 + \frac{1}{\hat{\nu}_{n,j}^2} \frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}}} \right). \tag{A69}
\]

Note that

\[
\sqrt{1 + \frac{1}{\hat{\nu}_{n,j}} \frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}}} = \frac{A^n \check{\tau}^n}{(\check{\tau}^n)^2} \sqrt{1 + \frac{1}{A^n \check{\tau}^n \left( x - 1 + e^{-x} \right) (1 - (1 + x)e^{-x})}} \leq \frac{\sqrt{A^n \check{\tau}^n}}{(\check{\tau}^n)^2} \sqrt{A^n \check{\tau}^n + \frac{1}{x - 1 + e^{-x}}} (1 - (1 + x)e^{-x}) \leq \frac{\sqrt{A^n \check{\tau}^n}}{(\check{\tau}^n)^2} \sqrt{1 + \frac{1}{x - 1 + e^{-x}}} (1 - (1 + x)e^{-x}) \tag{A70}
\]

where \( x = (p_1 + \gamma p_2)\check{\tau}^n \). We observe that

\[
g(x) = \sqrt{1 + \frac{1}{x - 1 + e^{-x}}} (1 - (1 + x)e^{-x}) \tag{A71}
\]

is a unimodal function on \( x \geq 0 \), reaching the maximum of approximately 1.08 at \( x = 5.8 \). Thus, \( 2g(x) \leq 2.16 \) for any non-negative \( x \), and the last expression in (A69) can be bound as follows

\[
= \frac{1}{2 \sqrt{\hat{\nu}_{n,j} + \frac{\hat{\nu}_{n,j}}{\hat{\nu}_{n,j}}} + \frac{1}{\hat{\nu}_{n,j}}} \left( \frac{A^n \check{\tau}^n - 1}{(\check{\tau}^n)^2} \right) - \frac{k}{\sqrt{k+1}} + 2 \sqrt{1 + \frac{1}{\hat{\nu}_{n,j}^2} \frac{\partial \check{\sigma}_{n,j}}{\partial \check{\nu}_{n,j}}} \right).
\]
Bavafa, Savin, Terwiesch: Managing Patient Panels

\[ \frac{1}{2\sqrt{\hat{r}_{ij} + \frac{1}{\pi^2}}} \left( \left( A^n \tau^n - 1 \right) \left( k \sqrt{k + 1} \right) + 2.16 \sqrt{A^n \tau^n} \right). \]  

(A72)

The expression inside the brackets in the last line above is non-positive for all

\[ (A^n \tau^n - 1) \left( k \sqrt{k + 1} \right) + 2.16 \sqrt{A^n \tau^n} \leq 0. \]  

(A73)

Using \( K = \frac{-2.16 + \sqrt{2.16^2 + 4 \left( \sqrt{\frac{k}{k + 1}} \right)^2}}{\frac{2}{\sqrt{k + 1}}} \), (A73) becomes

\[ \sqrt{A^n \tau^n} \leq K, \]  

(A74)

or

\[ p_1 + \gamma p_2 \geq \frac{\sqrt{p_1 p_2 \tau^n}}{K}, \]  

(A75)

which is equivalent to

\[ \gamma \geq \frac{1}{K} \sqrt{\tau^n \left( \frac{p_1}{p_2} \right) - \frac{p_1}{p_2}}. \]  

(A76)

Note that because of (14), \( k \geq 2 \). Thus,

\[ \frac{1}{K} \leq 1 + \frac{5}{3} \left( \frac{\sqrt{k + 1}}{k} \right). \]  

(A77)

Using (A77), we simplify (A76) to the following

\[ \gamma \geq \left( 1 + \frac{5}{3} \left( \frac{\sqrt{k + 1}}{k} \right) \right) \sqrt{\tau^n \left( \frac{p_1}{p_2} \right) - \frac{p_1}{p_2}}. \]  

(A78)

Note that if \( \gamma \) satisfies (A78), it also satisfies (A68).

For the results in part 2) of the Proposition, note that a fee-for-service physician’s objective function can be expressed using (A61):

\[ \hat{\Pi}_{FFS}^n = s - z_3 \sqrt{\hat{N}_{FFS}^n \hat{r}_{U,FFS}^n \left( 1 - \frac{\tau^n}{\tau} \right) \hat{N}_{FFS}^n \hat{r}_{U,FFS}^n}. \]  

(A79)

A fee-for-service physician is incentivized to choose the lowest RVI if

\[ \frac{d \left( -z_3 \sqrt{\hat{N}_{FFS}^n \hat{r}_{U,FFS}^n + \left( 1 - \frac{\tau^n}{\tau} \right) \hat{N}_{FFS}^n \hat{r}_{U,FFS}^n} \right)}{d \hat{r}_{FFS}^n} \leq 0. \]  

(A80)

Assuming that (18) holds, we have \( \frac{d (\hat{N}_{FFS}^n \hat{r}_{U,FFS}^n)}{d \hat{r}_{FFS}^n} \geq 0 \) as shown in part 1). Then,

\[ \frac{d \left( -z_3 \sqrt{\hat{N}_{FFS}^n \hat{r}_{U,FFS}^n + \left( 1 - \frac{\tau^n}{\tau} \right) \hat{N}_{FFS}^n \hat{r}_{U,FFS}^n} \right)}{d \hat{r}_{FFS}^n}. \]  

(A81)
\[
\begin{align*}
&= \left(\frac{-z_\delta}{2\sqrt{\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}}} \left( d\left(\frac{\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}}{\hat{r}^n_{FFS}}\right) + \left(\frac{1 - \tau^n}{\tau^n}\right) \frac{d:\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}}{d\hat{r}^n_{FFS}}\right) \right) \\
&= \frac{d\left(\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}\right)}{d\hat{r}^n_{FFS}} \left( \frac{-z_\delta}{2\sqrt{\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}}} + \left(\frac{1 - \tau^n}{\tau^n}\right) \right). \quad (A81)
\end{align*}
\]

As (A63) implies,
\[
\sqrt{\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}} = \frac{2s}{z_\delta} \left(\sqrt{1 + \frac{1}{k} \left(\frac{1}{\hat{r}^n_{FFS}} + \frac{\hat{\pi}^n_{U,FFS}}{\hat{\pi}^n_{U,FFS}}\right)} + 1\right). \quad (A82)
\]

Thus, (A81) becomes
\[
\frac{d\left(\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}\right)}{d\hat{r}^n_{FFS}} \left( -\frac{1}{k} \left(\sqrt{1 + k \left(\frac{1}{\hat{r}^n_{FFS}} + \frac{\hat{\pi}^n_{U,FFS}}{\hat{\pi}^n_{U,FFS}}\right)} + 1\right) + \left(\frac{1 - \tau^n}{\tau^n}\right) \right), \quad (A83)
\]

where \( k = \frac{4s}{z_\delta} \). Note that
\[
\left(\frac{1}{\hat{r}^n_{FFS}} + \frac{\hat{\pi}^n_{U,FFS}}{\hat{\pi}^n_{U,FFS}}\right) = 1 + \frac{1}{\hat{r}^n_{FFS} \hat{\pi}^n_{U,FFS}} \geq 1. \quad (A84)
\]

Thus,
\[
-\frac{1}{k} \left(\sqrt{1 + k \left(\frac{1}{\hat{r}^n_{FFS}} + \frac{\hat{\pi}^n_{U,FFS}}{\hat{\pi}^n_{U,FFS}}\right)} + 1\right) + \left(\frac{1 - \tau^n}{\tau^n}\right) \\
\leq -\frac{1}{k} \left(\sqrt{1 + k} + 1\right) + \left(\frac{1 - \tau^n}{\tau^n}\right). \quad (A85)
\]

The last expression is non-positive if and only if
\[
\tau^n \geq \frac{k}{k + 1 + \sqrt{k+1}}. \quad (A86)
\]

Next, we will derive a sufficient condition for
\[
\frac{d\left(-z_\delta \sqrt{\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}} + \left(\frac{1 - \tau^n}{\tau^n}\right) \hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}\right)}{d\hat{r}^n_{FFS}} \geq 0. \quad (A87)
\]

Suppose that (18) holds, so that \( \frac{d\left(\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}\right)}{d\hat{r}^n_{FFS}} \geq 0 \) as shown in part 1. Then,
\[
\frac{d\left(-z_\delta \sqrt{\hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}} + \left(\frac{1 - \tau^n}{\tau^n}\right) \hat{N}^n_{FFS} \hat{\pi}^n_{U,FFS}\right)}{d\hat{r}^n_{FFS}}
\]
\[
\frac{d}{d\hat{n}_{FFS}} \left( \frac{-z_d}{2\sqrt{\hat{n}_{FFS}} \hat{n}_{U,FFS}} + \left(1 - \tau^n\right) \right) = \frac{A88}{62}.
\]

From (A63) we get
\[
\sqrt{\hat{n}_{FFS}} \hat{n}_{U,FFS} = \frac{2s}{z_d \left(1 + \frac{4s}{z_d} \frac{1 + \hat{n}_{FFS}}{\hat{n}_{U,FFS}} + 1\right)}. \quad (A89)
\]

Note that
\[
\left(\frac{1}{\hat{n}_{FFS}} + \hat{n}_{U,FFS}\right) = 1 + \frac{1}{\hat{n}_{U,FFS}}. \quad (A90)
\]

Further,
\[
\hat{n}_{U,FFS} \hat{n}_{FFS} = \frac{\tau^n p_1 p_2 \hat{n}_{FFS}}{p_1 + \gamma p_2} \left(1 - \frac{1 - e^{-(p_1 + \gamma p_2) r_{FFS}^n}}{(p_1 + \gamma p_2) r_{FFS}^n}\right) = \tau^n A^n \left(y - 1 + e^{-y}\right). \quad (A91)
\]

where \(A^n = \frac{p_1 p_2}{(p_1 + \gamma p_2)}\), \(y = (p_1 + \gamma p_2) r_{FFS}^n\), \(y_u = (p_1 + \gamma p_2) r_u^n\), and \(r_u^n\) is an unconstrained minimum of (5) for \(m = n\) and \(c(1 + \Delta)\), satisfying:
\[
-1 - (1 - y_u^1) = 1 - \frac{1}{(1 + c(1 + \Delta)) A^n} = 0. \quad (A92)
\]

Thus, \(y_u\) satisfies
\[
y_u + 1 e^{-y_u} = 1 - \frac{1}{(1 + c(1 + \Delta)) A^n}. \quad (A93)
\]

Consider \(y_u^1\) satisfying
\[
y_u^1 + 1 \left(1 - y_u^1\right) = 1 - \frac{1}{(1 + c(1 + \Delta)) A^n}, \quad (A94)
\]

or
\[
y_u^1 = \frac{1}{\sqrt{(1 + c(1 + \Delta)) A^n}} \leq 1, \quad (A95)
\]

where the last inequality holds because using (17)
\[
c(1 + \Delta) > c(1 - \Delta) > \frac{1}{A^n} - 1. \quad (A96)
\]

Note that since \(e^{-y_u} \geq 1 - y_u\), \(y_u^1 \leq y_u\), and
\[
y_u - 1 + e^{-y_u} \geq y_u + 1 - y_u^1 \geq y_u^1 - 1 + 1 - y_u^1 + \frac{1}{3} (y_u^1)^2 = \frac{1}{3} (y_u^1)^2, \quad (A97)
\]

where \(e^{-x} \geq 1 - x + \frac{x^2}{2}\) applies to all \(x \in [0, 1]\).

Using (A95) and (A97) in (A91), we get
\[
\hat{n}_{U,FFS} \hat{n}_{FFS} \geq \tau^n A^n \frac{1}{3} (y_u^1)^2 = \frac{\tau^n}{3(c(1 + \Delta) + 1)}. \quad (A98)
\]
Thus,

\[
\left( \frac{1}{\hat{r}^n_{FFS}} + \hat{\pi}^n_{U,FFS} \right) \frac{\hat{\pi}^n_{U,FFS}}{\hat{r}^n_{FFS}} = 1 + \frac{1}{\hat{\pi}^n_{U,FFS}} \hat{r}^n_{FFS} \\
\leq 1 + \frac{3(c(1 + \Delta) + 1)}{\tau^n}.
\]  
\tag{A99}

Then,

\[
\frac{1}{2\sqrt{N^n_{FFS} \hat{\pi}^n_{U,FFS}}} \leq \frac{4s}{\left( \sqrt{1 + k \left( 1 + \frac{3(c(1 + \Delta) + 1)}{\tau^n} \right) + 1} \right) - \left( \sqrt{1 + \frac{3(c(1 + \Delta) + 1)}{\tau^n}} \right)}.
\]
\tag{A100}

Thus, the right-hand side of (A88) is non-negative when

\[
- \left( \frac{1}{2\sqrt{N^n_{FFS} \hat{\pi}^n_{U,FFS}}} \leq \frac{4s}{\left( \sqrt{1 + k \left( 1 + \frac{3(c(1 + \Delta) + 1)}{\tau^n} \right) + 1} \right) - \left( \sqrt{1 + \frac{3(c(1 + \Delta) + 1)}{\tau^n}} \right)} \right) \geq 0,
\]
\tag{A101}

or, equivalently, when

\[
\tau^n \leq \frac{k}{k + 1 + \frac{3(c(1 + \Delta) + 1)}{2} + \sqrt{\left( 1 + \frac{3(c(1 + \Delta) + 1)}{2} \right)^2 + k(1 + 3(c(1 + \Delta) + 1))}}.
\]
\tag{A102}

**Proof of Proposition 6**

We first show the results regarding the RVI values. In particular, suppose that (15), (16) and (18) hold, and let \( \Delta^n_r \) denote the value of \( \Delta \) for which \( r^n_r = r^r_+ \). For part 1) of Proposition 6, we establish that, for a capitation physician,

\[
\hat{r}^n_{CAP} \leq \hat{r}^r_{CAP}.
\]
\tag{A103}

For part 2) of Proposition 6, we show that, for a fee-for-service physician, if, in addition to (15), (16), and (18), either (19) or \( \Delta \leq \Delta^n_r \) hold, then

\[
\hat{r}^n_{FFS} \leq \hat{r}^r_{FFS},
\]
\tag{A104}

and if, in addition to (15), (16), and (18), (20) holds, then

\[
\exists \Delta_r, \gamma_r \in [0, 1], \text{ such that } \hat{r}^n_{FFS} \geq \hat{r}^r_{FFS}, \text{ for } \Delta \geq \Delta^n_r \text{ and } \gamma \geq \gamma_r.
\]
\tag{A105}
Regarding patient health, we show the following results. Suppose that (15), (16), and (18) hold, and let \( \Delta_\pi^n \) denote the value of \( \Delta \) for which \( \pi_U^n(r^n_+) = \pi_U^n(r^-) \). In part 1) of Proposition 6, we would like to show that, for a capitation physician,

\[
\hat{\pi}_{U,CAP}^n \leq \hat{\pi}_U^{t,CAP}.
\]

For part 2) of Proposition 6, we establish that, for a fee-for-service physician, if, in addition to (15), (16), and (18), either (19) or \( \Delta \leq \Delta^n_\pi \) hold, then

\[
\hat{\pi}_{U,FFS}^n \leq \hat{\pi}_U^{t,FFS},
\]

and, if, in addition to (15), (16), and (18), (20) holds, then

\[
\exists \Delta^n_x, \gamma_x \in [0, 1], \text{ such that } \hat{\pi}_{U,FFS}^n \geq \hat{\pi}_U^{t,FFS}, \text{ for } \Delta \geq \Delta^n_x \text{ and } \gamma \geq \gamma_x.
\]

First, we would like to show that \( \frac{\partial r^n_o}{\partial \gamma} \geq 0 \). Note that \( r^n_o \) satisfies the first order necessary condition

\[
f^n = (1 + c) \frac{\partial \pi_U^n}{\partial r^n_o} - \frac{1}{(r^n_o)^2} = 0,
\]

where

\[
\frac{\partial \pi_U^n}{\partial r^n_o} = \frac{A^n}{(r^n_o)^2} \left( 1 - ((p_1 + \gamma p_2)r^n_o + 1)e^{-(p_1 + \gamma p_2)r^n_o} \right),
\]

and \( A^n = \frac{p_1 p_2}{(p_1 + \gamma p_2)^2} \).

From (A109) it follows that \( \frac{df^n}{d\gamma} = 0 \), or, equivalently,

\[
\frac{\partial r^n_o}{\partial \gamma} = \frac{\partial f^n}{\partial \gamma}.
\]

Note that

\[
\frac{\partial f^n}{\partial r^n_o} = (1 + c) \frac{\partial^2 \pi_U^n}{\partial (r^n_o)^2} + \frac{2}{(r^n_o)^3}
\]

\[
= \frac{1}{(r^n_o)^2} \frac{\partial^2 \pi_U^n}{\partial (r^n_o)^2} + \frac{2}{(r^n_o)^3} - \frac{1}{(r^n_o)^2} \frac{\partial \log \left( \frac{\partial \pi_U^n}{\partial r^n_o} \right)}{\partial r^n_o} + \frac{2}{(r^n_o)^3},
\]

where we have used (A109). Further,

\[
\frac{\partial f^n}{\partial r^n_o} = \frac{1}{(r^n_o)^2} \frac{\partial \log \left( \frac{\partial \pi_U^n}{\partial r^n_o} \right)}{\partial r^n_o} + \frac{2}{(r^n_o)^3}
\]

\[
= \frac{1}{(r^n_o)^2} \left( \frac{\partial \log \left( 1 - ((p_1 + \gamma p_2)r^n_o + 1)e^{-(p_1 + \gamma p_2)r^n_o} \right)}{\partial r^n_o} \right) - \frac{2}{(r^n_o)^3} \frac{d \log r^n_o}{dr^n_o} + \frac{2}{(r^n_o)^3}.
\]
\[
\frac{\partial f_n}{\partial \gamma} = p_1 p_2 (1 + c) \frac{d}{dx} \left( \frac{1 - (x + 1)e^{-x}}{x^2} \right) \frac{\partial x}{\partial r_0^n},
\]

Further,

\[
\frac{d}{dx} \left( \frac{1 - (x + 1)e^{-x}}{x^2} \right) = \frac{e^{-x} ((x + 1)^2 + 1) - 2}{x^3},
\]

so that

\[
\frac{\partial f_n}{\partial \gamma} = \left( \frac{p_1 p_2}{A^n (1 - (x + 1)e^{-x})} \right) \left( \frac{e^{-x} ((x + 1)^2 + 1) - 2}{x^3} \right) (p_2 r_0^n)
\]

\[
= p_2 (p_1 + \gamma p_2) \left( \frac{e^{-x} ((x + 1)^2 + 1) - 2}{x^2 (1 - (x + 1)e^{-x})} \right),
\]

where we have used (A109). Then, as it follows from (A111),

\[
\frac{\partial r_0^n}{\partial \gamma} = p_2 (p_1 + \gamma p_2) \left( \frac{2e^{-x} - ((x + 1)^2 + 1)}{x^3} \right) \geq 0,
\]

so that patients respond to using non-physician providers by decreasing the values of both \( r_- \) and \( r_+ \). As it follows from part 1) of Proposition 5, a capitation physician will choose the RVI value equal to \( r_+ \) as long as (18) holds, so that \( r_0^n \leq r_+ \). This established the result for part 1) of this Proposition regarding RVI values.

On the other hand, as it follows from part 2) of Proposition 5, under (18) and (19), a fee-for-service physician will choose the RVI value equal to \( r_- \), and, since \( r_0^n \leq r_- \), we have \( r_{FFS}^n \leq r_{FFS}^n \) since \( r_+ \leq r_+ \). This established the result for part 2) of this proposition regarding RVI values (for the case where (19) holds).

Finally, from part 2) of Proposition 5 it follows that, under (18) and (20), a fee-for-service physician will choose the RVI value equal to \( r_+ \). Thus, the optimal RVI decreases if and only if \( r_+ \leq r_- \). Note that if \( \Delta = 0 \), then \( r_+ \leq r_+ = r_- \). Thus, there exists \( \Delta_+^n \) such that \( r_+ \leq r_+ \) for all \( \Delta \leq \Delta_+^n \). On the other hand, if \( \Delta \rightarrow 1 \), then \( r_+ \geq r_- \). In this case, if \( \gamma = 1 \), then \( r_+ = r_+ \geq r_- \). Thus, there exist \( \Delta_-^n \) and \( \gamma_- \) such that \( r_+ \geq r_- \) for all \( \Delta \geq \Delta_-^n \) and \( \gamma \geq \gamma_- \). This established the results of part 2) regarding RVI values (for the case where \( \Delta \leq \Delta_+^n \) holds and the case where (20) holds).

So far we proved the results for RVI values. Next, we prove the results for patient health.
First, we want to show that \( \frac{d\pi^n_U}{d\gamma} > 0 \), where we denote \( \pi^n_U = \pi^n_U (\tau^n) \). Note that
\[
\frac{d\pi^n_U}{d\gamma} = \frac{\partial \pi^n_U}{\partial \gamma} + \frac{\partial \pi^n_U}{\partial r^n} \frac{dr^n}{d\gamma}.
\] (A118)

Further, using (4), we obtain
\[
\frac{\partial \pi^n_U}{\partial \gamma} = -A^n p_2 \left( 1 - \frac{1 - e^{-x}}{x} \right) + A^n p_2 \left( \frac{1 - e^{-x}}{x^2} \right),
\] (A119)

with \( x = (p_1 + \gamma p_2) r^n \) and \( A^n = \frac{p_1 p_2}{(p_1 + \gamma p_2)^2} \). Simplifying (A119), we get
\[
\frac{\partial \pi^n_U}{\partial \gamma} = A^n p_2 \left( 2 \left( \frac{1 - e^{-x}}{x} \right) - (1 + e^{-x}) \right).
\] (A120)

Also, using (4)
\[
\frac{\partial \pi^n_U}{\partial r^n} = p_1 p_2 \left( \frac{1 - (x + 1) e^{-x}}{x^2} \right).
\] (A121)

Finally, combining (A119), (A121) and (A117), we have
\[
\frac{d\pi^n_U}{d\gamma} = A^n p_2 \left( 2 \left( \frac{1 - e^{-x}}{x} \right) - (1 + e^{-x}) \right) + p_1 p_2 \left( \frac{1 - (x + 1) e^{-x}}{x^2} \right) p_2 (r^n)^2 \left( \frac{2e^x - ((x + 1)^2 + 1)}{x^3} \right)
\]
\[
= A^n p_2 \left( 2 \left( \frac{1 - e^{-x}}{x} \right) - (1 + e^{-x}) + (1 - (x + 1) e^{-x}) \left( \frac{2e^x - ((x + 1)^2 + 1)}{x^3} \right) \right)
\]
\[
= A^n p_2 e^{-x} \left( 2x^2 (e^x - 1) - x^3 (e^x + 1) + (e^x - (x + 1)) \left( 2e^x - ((x + 1)^2 + 1) \right) \right).
\] (A122)

The expression inside the brackets in (A122) is positive for all positive \( x \), so that \( \frac{d\pi^n_U}{d\gamma} > 0 \). Thus, parts 1) and 2) (for the case where (19) holds) of this Proposition regarding patient health follow from parts 1) and 2) of Proposition 5.

Based on part 2) of Proposition 5, we observe that, under (18) and (20), a fee-for-service physician will choose the RVI value equal to \( r^\_ \). If, after the introduction of non-physician providers, \( r^n > r^\_ \), \( \tilde{\pi}_{U,FFS} \) increases. If \( \Delta \rightarrow 1 - \frac{1}{\tilde{r}^\_} \left( \frac{(p_1 + \gamma p_2)^2}{p_1 p_2} \right) - 1 \) we have \( r^\_ \rightarrow \infty \), but \( r^\_ \) is finite. In this case, if \( \gamma = 1 \), then \( r^n = r^n = r^\_ \) increases. Thus, there exist \( \Delta^n \) and \( \gamma^n \) such that \( r^n \geq r^\_ \) for all \( \Delta \geq \Delta^n \) and \( \gamma \geq \gamma^n \).

On the other hand, for \( \Delta = 0 \), we have \( r^n = r^n \). Therefore, \( r^n \leq r^\_ \), and \( \tilde{\pi}_{U,FFS} \) decreases. This outcome remains valid for values of \( \Delta \) sufficiently close to 0. This established the results for part 2) of this Proposition regarding patient health (for the case where \( \Delta \leq \Delta^n \) holds and the case where (20) holds).

**Proof of Proposition 7**

Suppose that (15) and (16) hold. For part 1) of Proposition 7, we show that, for a capitation physician, if, in addition, (18) holds, then there exists \( \tilde{\tau}^{n\_}_{CAP} \in [0, 1] \) such that for any \( \tau^n \geq \tilde{\tau}^{n\_}_{CAP} \), there exists \( \tilde{\gamma}_{CAP} (\tau^n) \in [0, 1] \) such that
\[
\tilde{\chi}_{CAP} (\tau^n) \geq \tilde{\gamma}_{CAP} (\tau^n) \iff \gamma \geq \tilde{\gamma}_{CAP} (\tau^n).
\] (A123)
where $\frac{\partial \tau_n}{\partial \pi_j} \geq 0$. Also, if $\tilde{\tau}_n > 0$, then
\[
\hat{N}_C A P \geq \hat{N}_C A P^t, \text{ if } \tau^n < \tilde{\tau}_C A P. \tag{A124}
\]

Next, we show that for a fee-for-service physician, if, in addition, (18) and (19) hold, then there exists $\tilde{\tau}_{FFS} \in [0, 1]$ such that for any $\tau^n \geq \tilde{\tau}_{FFS}$, there exists $\gamma^n_{FFS}(\tau^n) \in [0, 1]$ such that
\[
\hat{N}_{FFS} \geq \hat{N}_{FFS}^t \leftrightarrow \gamma \geq \gamma_{FFS}(\tau^n), \tag{A125}
\]
where $\frac{\partial \tilde{\gamma}_{FFS}}{\partial \tau^n} \geq 0$. Also, if $\tilde{\tau}_{FFS} > 0$, then
\[
\hat{N}_{FFS} \geq \hat{N}_{FFS}^t, \text{ if } \tau^n < \tilde{\tau}_{FFS}. \tag{A126}
\]

We also establish that, for a fee-for-service physician,
\[
\hat{\Pi}_{FFS} \geq \hat{\Pi}_{FFS}^t. \tag{A127}
\]

We start by observing that the expression (A63) for $\hat{N}_j^n$, $j = FFS, CAP$, only contains $\hat{\pi}_{U,j}^n$ in its denominator. Note that the denominator is an increasing function of $\hat{\pi}_{U,j}^n$ and $\frac{\partial \hat{\pi}_{U,j}^n}{\partial \tau^n} > 0$. Therefore,
\[
\frac{\partial \hat{N}_j^n}{\partial \tau^n} < 0. \tag{A128}
\]

Next, we want to prove that, under (18) for a capitation physician and under (18) and (19) for a fee-for-service physician, $\frac{d \hat{N}_j^n}{d \gamma} > 0$. Note that
\[
\frac{d \hat{N}_j^n}{d \gamma} = \frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \left( \frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma} + \frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma} \right) + \frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma}
\]
\[
= \frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma} + \frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma}
\]
\[
\geq \frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma} + \frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma}, \tag{A129}
\]

We first show that $\frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \geq 0$ and then that $\frac{d \hat{N}_j^n}{d \gamma} \geq 0$.

To prove that $\frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \geq 0$ we show that $\frac{\partial \hat{N}_j^n}{\partial \gamma} \leq 0$ and $\frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma} \leq 0$. Note that from (A120), $\frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma} \leq 0$. Therefore, $\frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma} \leq 0$. Also, $\hat{\pi}_{U,j}^n$ appears only in the denominator of the $\hat{N}_j^n$ expression in (A63), and the denominator is an increasing function of $\hat{\pi}_{U,j}^n$, so that $\frac{\partial \hat{N}_j^n}{\partial \hat{\pi}_{U,j}^n} \leq 0$.

Next, we prove that $\frac{d \hat{N}_j^n}{d \gamma} \geq 0$ by showing that $\frac{d \hat{N}_j^n}{d \gamma} \geq 0$ and $\frac{\partial \hat{\pi}_{U,j}^n}{\partial \gamma} \geq 0$. Under (18), we have $\frac{\partial \tau^n}{\partial \pi_j} \geq 0$ according to part 1) of Proposition 6. Moreover, under (18) and (19), we have $\frac{\partial \tilde{\gamma}_{FFS}}{\partial \gamma} \geq 0$ according to part 2) of Proposition 6. Furthermore, in part 1) of Proposition 5 we proved that, under (18), $\frac{d \hat{N}_j^n}{d \gamma} \geq 0$. 


So far we have shown that, under (18) for a capitation physician and under (18) and (19) for a fee-for-service physician, $\hat{N}_j^n$ is a non-decreasing function of $\gamma$ and a decreasing function of $\tau^n$. Next, we want to show that $\tilde{\tau}_j^n$ and $\hat{\gamma}_j$ exist. Note that $\tau^n$ and $\gamma$ can take any value between 0 and 1.

Note that the case of $(\gamma, \tau^n) = (1, 1)$ corresponds to the traditional care mode. Also, for $(\gamma, \tau^n) = (0, 0)$, it is possible to have $\hat{N}_j^n \geq \hat{N}_j^t$ as well as $\hat{N}_j^n < \hat{N}_j^t$. In what follows, we show existence of $\tilde{\tau}_j^n$ and $\hat{\gamma}_j$ in either of these two cases.

First, suppose that $\hat{N}_j^n < \hat{N}_j^t$ for $(\gamma, \tau^n) = (0, 0)$. Then, we can select $\tilde{\tau}_j^n$ to be equal to 0. Next, pick an arbitrary $\tau^n$ between 0 and 1 and call it $\tau^n$. Because $\hat{N}_j^n$ is an increasing function of $\gamma$, to prove existence of $\hat{\gamma}_j$ we need to show that $\hat{N}_j^n < \hat{N}_j^t$ for high enough $\gamma$ and $\hat{N}_j^n < \hat{N}_j^t$ for low enough $\gamma$. Indeed, for $(\gamma, \tau^n) = (0, \tau^n)$ we have $\hat{N}_j^n < \hat{N}_j^t$ since $\hat{N}_j^n < \hat{N}_j^t$ at $(\gamma, \tau^n) = (0, 0)$ and $\frac{\partial \hat{N}_j^n}{\partial \tau^n} < 0$. Also, at $(\gamma, \tau^n) = (1, \tau^n)$ we have $\hat{N}_j^n \geq \hat{N}_j^t$ since $\hat{N}_j^n = \hat{N}_j^t$ at $(\gamma, \tau^n) = (1, 1)$ and $\frac{\partial \hat{N}_j^n}{\partial \gamma} < 0$. Thus, there exists $\hat{\gamma}_j$ such that $\hat{N}_j^n = \hat{N}_j^t$ for $(\gamma, \tau^n) = (\hat{\gamma}_j, \tau^n)$.

Now, suppose $\hat{N}_j^n > \hat{N}_j^t$ at $(\gamma, \tau^n) = (0, 0)$. Note that $\hat{N}_j^n < \hat{N}_j^t$ at $(\gamma, \tau^n) = (0, 1)$ since $\hat{N}_j^n = \hat{N}_j^t$ at $(\gamma, \tau^n) = (1, 1)$ and $\frac{\partial \hat{N}_j^n}{\partial \gamma} \geq 0$. Therefore, there exists $\tau^n = x$ such that $\hat{N}_j^n = \hat{N}_j^t$ at $(\gamma, \tau^n) = (0, x)$ because $\frac{\partial \hat{N}_j^n}{\partial \tau^n} < 0$. Note that $\hat{N}_j^n \geq \hat{N}_j^t$ at $(\gamma, \tau^n) = (0, y)$ for all $y < x$ because $\frac{\partial \hat{N}_j^n}{\partial \gamma} > 0$, so $\hat{N}_j^n \geq \hat{N}_j^t$ at $(\gamma, \tau^n) = (\gamma, y)$ for all $y < x$ and $\gamma \in [0, 1]$ since $\frac{\partial \hat{N}_j^n}{\partial \gamma} > 0$. This proves (A124) and (A126).

Set $\tilde{\tau}_j^n = x$ and pick an arbitrary $\tau^n$ between $\tilde{\tau}_j^n$ and 1 and call it $\tau^n$. Because $\hat{N}_j^n$ is an increasing function of $\gamma$, to prove existence of $\hat{\gamma}_j$ in this case we need to show that $\hat{N}_j^n > \hat{N}_j^t$ for high enough $\gamma$ and $\hat{N}_j^n < \hat{N}_j^t$ for low enough $\gamma$. Indeed, for $(\gamma, \tau^n) = (1, \tau^n)$ we have $\hat{N}_j^n \geq \hat{N}_j^t$ since $\hat{N}_j^n = \hat{N}_j^t$ at $(\gamma, \tau^n) = (1, 1)$ and $\frac{\partial \hat{N}_j^n}{\partial \gamma} < 0$. Also, at $(\gamma, \tau^n) = (0, \tau^n)$ we have $\hat{N}_j^n < \hat{N}_j^t$ since $\hat{N}_j^n = \hat{N}_j^t$ at $(\gamma, \tau^n) = (0, \tilde{\tau}_j^n)$ and $\frac{\partial \hat{N}_j^n}{\partial \tau^n} > 0$. Thus, there exists $\hat{\gamma}_j$ such that $\hat{N}_j^n = \hat{N}_j^t$ for $(\gamma, \tau^n) = (\hat{\gamma}_j, \tau^n)$. In summary, we have shown that $\hat{\gamma}_j$ and $\tilde{\tau}_j^n$ exist in all cases, proving the statements in part 1).

Next, we prove that $\frac{\partial \hat{\gamma}_j}{\partial \tau^n} \geq 0$. Suppose that we find $\hat{\gamma}_j(\tau^n)$ for a specific $\tau^n$ called $\tau^n = [\tilde{\tau}_j^n, 1)$, such that $\hat{N}_j^n = \hat{N}_j^t$ at $(\gamma, \tau^n) = (\hat{\gamma}_j, \tau^n)$. Then, for a small enough positive $\epsilon$, we have $\hat{\gamma}_j(\tau^n) < \tau^n + \epsilon \leq 1$, and $\hat{N}_j^n < \hat{N}_j^t$ for $(\gamma, \tau^n) = (\hat{\gamma}_j, \tau^n + \epsilon)$ since $\frac{\partial \hat{N}_j^n}{\partial \tau^n} < 0$. So, $\hat{\gamma}_j(\tau^n) \leq \hat{\gamma}_j(\tau^n + \epsilon)$ because $\frac{\partial \hat{N}_j^n}{\partial \tau^n} \geq 0$. Also, note that $\hat{\gamma}_j(\tau^n + \epsilon)$ exists since we proved earlier in this proposition that $\hat{\gamma}_j(\tau^n)$ exists for all $\tau^n \geq \tilde{\tau}_j^n$.

In the next step, we establish the result on the revenue of the fee-for-service physician for part 2) of this proposition. To see the effect of non-physician providers on the revenue of a fee-for-service physician, note that $\frac{\partial \hat{N}_j^n}{\partial x} < 0$ since $\frac{\partial \hat{N}_j^n}{\partial x} < 0$ and $\frac{\partial \hat{N}_j^n}{\partial x} = 0$. Then, based on (A79), the optimal revenue function value for a fee-for-service physician at $\tau^n = 1$ is

$$\hat{\Pi}_{FFS} = s - z_{\delta} \sqrt{\hat{N}_j^n \hat{\beta}_U^{FS}.}$$  (A130)
From (A63), the expression under the square root in (A130) is
\[
\sqrt{\frac{N_{FFS}^{\pi_u}}{\hat{N}_{FFS}^{\pi_u} \hat{\pi}_{FFS}}} = \frac{2s}{z_s \left(1 + \frac{4s}{\delta^2} \left(\frac{1}{r_{FFS}^{\pi_u} \hat{\pi}_{FFS}} + 1\right) + 1\right)}.
\] (A131)

Consider the case where a fee-for-service physician chooses \(r_u\), and denote the expected revenue earned by a physician under this policy with \(\tilde{\Pi}_{FFS}^{\pi_u}\). Because choosing \(r_u\) is not always optimal for a fee-for-service physician, we have \(\tilde{\Pi}_{FFS}^{\pi_u} \leq \hat{\Pi}_{FFS}^{\pi_u}\). Based on part 2) of Proposition 6, \(\frac{d\tilde{r}_{FFS}}{d\gamma} \geq 0\). Also, we showed that \(\frac{d\pi_u}{d\gamma} \geq 0\) in part 2) of Proposition 6. Therefore, \(\frac{d\left(\sqrt{\frac{N_{FFS}^{\pi_u} \pi_u \hat{\pi}_{FFS}}}{\hat{N}_{FFS}^{\pi_u} \hat{\pi}_{FFS}}\right)}{d\gamma} \leq 0\). Thus, even at \(\tau_u = 1\) the value of \(\tilde{\Pi}_{FFS}^{\pi_u}\) is a non-increasing function of \(\gamma\), so it has its lowest value at \(\gamma = 1\) which corresponds to the traditional care mode. Therefore, \(\hat{\Pi}_{FFS}^{\pi_u} \geq \tilde{\Pi}_{FFS}^{\pi_u} \geq \tilde{\Pi}_{FFS}^{t_u}\), and this proves the statement in part 2).